ENME 320: Fluid Mechanics

Turibius Rozario

August 30, 2023 – December 11, 2023

Contents

1 08/30 Introduction 1

2 09/01: Fluid dimensions and basics 2

3 09/08: Viscosity, Reynolds and mach number 4

4 09/11: Math and Thermodynamics Review 5

5 09/13: Hydrostatic Pressure 6

6 09/15: Changing Pressure 7

7 09/18: Bernoulli's Equation 8

8 09/20: Bernoulli Example Problems 9

9 09/22: Bernoulli with Curvature 10

10 09/25: Finishing up on Bernoulli's Equations 11

11 09/27: Final Examples and Fluid Kinematics 12

12 09/29: Exam Prep Review 14

13 10/03: Lines and Acceleration 14

14 10/06: Exam Results Review 15

15 10/09: Control Volumes and Systems 16

16 10/11: Control Volumes and Systems 17

17 10/13: Homework and Quiz Questions 18

18 10/16: Using Reynolds Transport Theorem 19

19 10/18: Using Reynolds Transport Theorem 20

20 10/20: Deforming Control Volumes 21

21 10/23: Conservation of Linear Momentum 22

22 10/25: More momentum 23

23 10/27: More momentum 24

24 10/30: Finishing linear momentum 24

25 11/01: Finishing Chapter 5 25

26 11/03: Midterm Prep 26

27 11/08: Differential Analysis 26

28 11/10: Continuing Differential Analysis 27

29 11/13: Stream Functions 28

30 11/15: Potential Flow and Navier-Stokes 30

31 11/22: Cylindrical Flow 31

32 11/27: Buckingham Pi 31

33 11/29: Boundary layers and airfoils 33

34 12/01: Airfoils, Lift, Drag, Stall 34

35 12/04: Supersonics 34

36 12/06: Shockwaves 35

37 12/11: Review 36

08/30 Introduction

We can call our professor Jorge (J is 'H'), a PhD candidate. We can feel free to go to his office hours, which is in the lab ENGR302, Monday and Wednesday 1200 to 1400. Email: `jorgeaa1@umbc.edu`. The syllabus is posted on BlackBoard. The professor covered the syllabus mostly.

Once Natalija Marin, the TA, sets up office hours, we will get an email.

We can opt-out of the textbook, since it is optional. We are recommended to have this book, but not necessarily this book. We can have any book, actually, but the book by Munson and Okiishi is recommended.

Homework has to be submitted before the next class (1 week). We will have about 6 or 7 quizzes. We can work in team and pairs for the quizzes.

If we need to schedule exams with SDS, we should do it 48 hours before.

Grading:

- 10% Homework
- 10% Quizzes
- 15% Projects
- 20% Exam 1
- 20% Exam 2
- 25% Final

The distribution of the grading may change, or there may be extra credit. The homework is expected to take the most time despite so little % grading; if we can master the homework, we can master the exams. We are allowed to work with each other in homework. The professor will try to make the exam look similar to the homework.

The podcast by Boston University is optional.

The numbers in the outline of the syllabus represent the lecture number. The lecture topic is tentative. The stuff underneath the lecture are the tasks we need to do before that lecture.

For Wednesday, September 6th, Natalia will be teaching; we will have a diagnostics quiz, that is meant to gauge our level. This will probably not be counted towards the end; or the professor may drop the lowest grade. Different homework will have different numbers of questions; each will be 10 points.

Masks are optional in the class.

The Academic Success Center may provide help. The syllabus contains additional information such as confidential services and for parenting students.

The project should be done early. It is due on Sunday, October 15th, at midnight. If the file is large, we can upload to Google Drive and share the link in BlackBoard. The project is regarding any phenomenon in our daily life related to fluid mechanics. We can work individually or in groups of four people. We have to record the phenomenon somehow, such as picture or drawing. We can combine our own photos with ones from the internet. If we do an experiment instead, especially with water, we should not waste more than 1 litre of water.

Later, we have to answer questions regarding the project. E.g. what it is, what are the conditions needed for this phenomenon, the theory that the observation relies on, etc. We need to make a visual presentation, such as a poster, PowerPoint presentation, etc. Make it logically flowing. Grading of this is:

- 40% visual aspects
- 60% content material

This is worth 7.5% of our total grade. We can get a 10% extra grade.

Jorge was previously a Masters student at New Mexico state. Previously had a dual degree. His PhD work is regarding the ignition of droplets. He uses electric fields or other forms of levitation. There are many phenomena in his experiments. Water would start aggregating on his alcohol droplets. As things evaporate, the droplets break apart. He did research on supersonic shock waves. If there is supersonic flow, there is not enough time for fuel to mix with the air.

History of fluid mechanics. Archimedes got into the theory of buoyancy. Volume of an object can be measured by the volume of water that the object displaces. Additional materials over the history were covered, such as the Spanish Armada, various 20th century developments, and many current developments. Many developments in 20th century starting with Wright brother's flight. CFD breaks up body in tiny parts and discretize the Navier-Stokes equations. Now lots of computations done by chips, which require a lot of cooling.

09/01: Fluid dimensions and basics

Last class, discussion was on history of fluid mechanics. In Roman empire, aqueducts were used to transport water; they had knowledge of pressure and height difference. We talked about other history, Bernoulli, Navier-Stokes equation, etc. The newest approach to studying fluid dynamics is computational fluid dynamics (CFD). The numerous bad weather we had over the summer is related with fluids, since they involve water and air.

Do's and don'ts for the class are shown on silde 25 of the introduction slides. Description of the project is already up on BlackBoard, but is not due until mid-semester.

Usually, homework will be posted on Friday, and made due for the next Friday.

Chapter 1 slides posted on BlackBoard. A fluid will deform continuously if shear is applied. As an example, if we try to apply shear on air, it will keep deforming/moving. We will cover fluid statics (when they don't move), fluid kinematics (their motion), and fluid dynamics (relation between forces and fluid flow).

Qualitative dimensions can be described using the Mass Length Time (MLT) system, or the Force Length Time (FLT) system. Time is denoted by T, while temperature is denoted by θ. Secondary dimensions for MLT:

- Area L^2
- Velocity LT^{-1}
- Density ML^{-3} (derived from density m/V)
- Force $F = MLT^{-2}$

For the FLT system, we have

- Mass FT^2L^{-1}
- Pressure FL^{-2}
- Energy FL

The mass and force relationship,

$$M = \frac{FT^2}{L} \tag{1}$$

can be used to switch between the different systems.

We need to maintain dimensions on both sides of equations when doing the calculations.

We will use SI system. Units:

- Length (meter)
- Time (second)
- Mass (kilogram)
- Force (Newton)
- Pressure (Pascal)
- Energy (Joule)
- Power (Watt)

Many units are related with water. For example, $1\,\mathrm{dm}^3 = 1\,\mathrm{litre}$. The other unit system is british gravitational:

- Length (foot)
- Second (s)
- Mass (slug)
- Force (pound)
- Pressure (psi)
- Energy (foot pound)
- Torque (pound foot)

Though not popular, it is possible to call mass as 'mass pound'. 1 pound of force on 1 mass pound yields $1g$ acceleration.

List of fluid properties:

- Density $\rho = \frac{m}{V}$
- Specific volume $v = \frac{V}{m} = \frac{1}{\rho}$
- Specific weight $\gamma = \frac{\text{weight}}{V} = \rho g$; this is basically density scaling with gravity.
- Specific gravity $SG_{liq} = \frac{\rho_{liq}}{\rho_{H_2O@4^\circ C}}$ or $SG_{gas} = \frac{\rho_{gas}}{\rho_{air@1atm,20^\circ C}}$.

Specific gravity is dimensionless. Mercury, a liquid metal at room temperature, has a very large specific gravity of 13.55. We can determine the density of mercury by using this number and the specific gravity of water.

Pressure is force per unit area exerted on a real or imaginary plane. There are a lot of molecules of air, hitting us everywhere and all the time. They have very low density, but there is a lot, so air has a pressure. There is a different pressure between the floor and ceiling; this would be more apparent if the room was filled with water.

Two ways to measure pressure:

1. Absolute pressure: Pressure compared to perfect vacuum. Absolute zero pressure is in perfect volume. Sea-level pressure is 101.33 kPa (absolute).
2. Gage pressure: Pressure relative to local atmosphere (typically).

As a result, the gage pressure of air is 0 kPa, because we are comparing it against itself. In perfect volume, gage pressure is −101.3 kPa.

The heart has movements. Systolic movement is generally 120 mm Hg, which is equivalent to 16 kPa (relative to air). Absolute pressure is like the Kelvin scale. Atmospheric pressure high up is negative (gage). The (gage) pressure of our heart becomes very high if we were to go up in the mountains.

Temperature is the way to measure kinetic energy of the particles. It is an average, since there are a lot of particles. Using the hypothetical case of no molecule movement, Kelvin scale made. But absolute zero cannot practically happen, because then the particle would collapse. 1 K = 1.8 °R. Relative temperatures are ones like Celsius and Fahrenheit.

Gasses are very sensitive to changes in pressure and temperature. Ideal gas equation:

$$p\forall = n\hat{R}T \tag{2}$$

where p is absolute pressure, $\forall$ is volume, n is moles of gas, T is absolute temperature, and $\hat{R} = 8.31\,\mathrm{JK^{-1}mol^{-1}}$ which is the universal gas constant. Another formula used is

$$\rho = \frac{p}{RT} \tag{3}$$

where $R = \hat{R}/M$.

Viscosity is fluidity, how likely it is to flow, given by μ. Honey opposes more to flow. How much shear stress that have to be applied to the fluid to make it move is the viscosity:

$$\tau = \mu \frac{\partial u}{\partial y} \tag{4}$$

No-slip condition is that next to a solid surface, the velocity of the fluid relative to that surface is zero. In figure 1, the top plate is moving with velocity U,

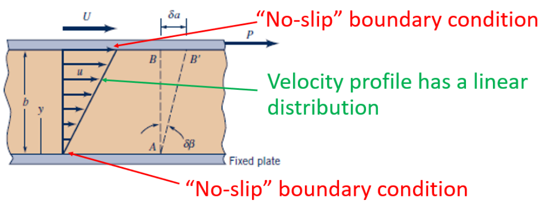

Figure 1: Explanation for Viscosity

whereas the bottom is not. The triangle is the velocity profile. The small u is the velocity at each location, while y is the vertical location shown in the figure. Since this is a linear distribution:

$$\frac{\partial u}{\partial y} = \frac{U}{b} \tag{5}$$

Using the shear stress and the above result, we can compute μ, which is the dynamic or absolute viscosity.

09/08: Viscosity, Reynolds and mach number

Purpose of the diagnostics quiz last class (no lecture) was to see where we are in math and thermodynamics. We will have a review of some thermodynamics and math. Three approaches to study fluid mechanics are: theoretical, computational, and experimentally. Generally, fluid flow is indicated going left to right, so fishes are supposed to be going right.

Homework 2 will be due Monday, 18th.

Last class, fluid properties, particularly viscosity μ were discussed. More viscosity of fluid needs more force to make it flow.

$$\tau = \frac{F}{A} = \mu \frac{\partial u}{\partial y} \tag{6}$$

where u is the velocity along the pipe and y is the position from the fixed point. The velocity gradient is given by the maximum velocity and diameter, U/b.

For a mach number less than 0.3, the fluid properties hold true. Equation 6 is for Newtonian fluids; the graph of this is linear. However, shear could be thickening or thinning, such as Oobleck and quick sand. Bingham plastic, such as toothpaste, is where applying shear stress does not make it move, but after a threshold it will. Shear thickening example is shampoo, which won't run away when on our hand. Toothpaste is neither solid nor fluid, since it has both properties depending on shear stress.

A viscometer measures viscosity by spinning at a given frequency, and providing different shearing stress. Temperature can also be adjusted.

Liquids are governed by the Andrade's equation

$$\mu_{\text{liquid}} = De^{\frac{B}{T}} \tag{7}$$

where T is the temperature. As an example, cold honey won't pour, but if we warm it, it will flow. Gasses are the opposite

$$\mu_{\text{gas}} = \frac{CT^{\frac{3}{2}}}{T+S} \tag{8}$$

This is because of how gasses and liquids work differently. Heating liquid breaks their forces. For gasses, increasing temperature increases pressure, which ends up increasing the collision.

For fluid flow in a pipe, velocity is zero on the walls, and maximum at the center, and the distribution is not uniform. The equation of the velocity is given by

$$u = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right] \tag{9}$$

The above equation is for two parallel plates. A figure of non-linear distribution is shown in figure 2.

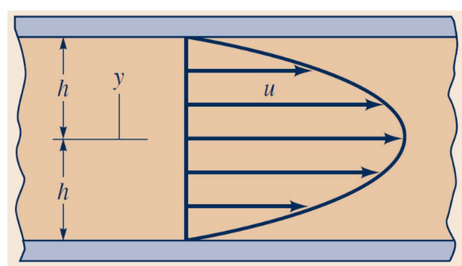

Figure 2: Non-linear velocity distribution for Newtonian fluids

Maximum shear stress will be on the walls.

Reynolds number is given by

$$\text{Re} = \frac{\rho V D}{\mu} \tag{10}$$

where ρ is density, V is velocity, and D is the diameter of the pipe. It tells us how much the fluid moves in contrast to how much it can move, i.e., how much it wants sto move. This is dimensionless.

Speed of sound/propagation is the way fluid communicates change. Applying change to one end of a fluid will be delivered to the other end at the speed of sound. For example, when we close a tap, there is high pressure immediately at the tap, and be transferred to the other end of the piping.

The speed of sound will be

$$c = \sqrt{\gamma R T} \tag{11}$$

where R is the ideal gas constant, T is temperature, and γ is the ratio of specific heat of constant pressure and volume.

Mach number is given by

$$M = \frac{u}{c} \tag{12}$$

Regular fluid equations work until about 0.3 mach. When something moves at the speed of sound, all the sound waves seemingly compresses right in front of the something. Drag, analytically, increases to infinite, but realistically does not; different kind of equations have to be used for mach numbers greater than 1. When the object flies faster than the speed of sound, its own sound is left behind.

Water is always evaporating. Sometimes the underneath water molecules bump the top water into the air. The gas molecules can, also, condense back to the water. This always happens. An open water container will evaporate completely.

A closed container, in contrast, will reach equilibrium for evaporation. The pressure provided by the evaporated water on the walls of the closed container is the vapor pressure. Each fluid have their own different vapor pressure.

Evaporate happens continuously; boiling happens at the boiling point, and bubbling can be seen. Water can turn from liquid to gas by heating, or by decreasing pressure. This creates cavitations (bubbling). When the bubble collapses, it creates a jet, that causes damage. The pistol shrimp shuts its claw to create a bubble that bursts and creates high pressure/temperature.

Surface tension is the membrane resulting from unbalanced forces that molecules have on each other. In the liquid-air boundary, there is no liquid pulling the liquid into the air, hence the liquid sticks to liquid. These forces become very important in the micro and nano scale. The Marangoni effect is another example, where fluid can be moved by changing surface tension (e.g. putting soap/alcohol into water breaks water's surface tension). Water has one of the highest surface tension; soap breaks this down, hence can clean.

09/11: Math and Thermodynamics Review

Review on math and thermodynamics based on quiz results. Partial derivatives would be brushed over only quickly.

The universe is the totality, everything there is. A thermodynamic system is a part of the universe, a certain amount of fixed mass; the change of mass in a system is zero:

$$\frac{Dm_{sys}}{Dt} = 0 \tag{13}$$

The capital D indicates the 'total derivative'.

The system boundary is the boundary that separates the threshold of mass from the universe; this can be real or imaginary. In a closed system, mass can not enter/exit, but energy can. In an isolated system, energy is not transferred either.

The state of a system can be changed from one to another via a process; the process can be extensive or intensive. Extensive relates to the amount of mass, whereas intensive does not.

- mass — extensive
- volume — extensive
- pressure — intensive
- temperature — intensive
- density — intensive
- weight — extensive

Pressure is intensive because both mass and volume change with mass, therefore the property of pressure becomes intensive. Volume is extensive, but can be made intensive, such as the specific volume. Gas volume in a closed container will remain an extensive property, because we can take a smaller section of that gas, and we'll get a *different* volume.

In thermodynamics, the average effect of each particle is considered; for example, the temperature is the *average* kinetic energy of all of the molecules. This assumption does not always work.

The continum model holds true as long as the following relationship is true:

$$\frac{\lambda}{l} << 1 \tag{14}$$

where λ is the Knudssen number, and l is the characteristic length. For example, the diameter of a chimney can be l, and the space between two molecules is λ. However, really high up in space, because gas becomes so far apart, λ becomes large and then we'll have to consider the effect of individual atoms. In this class, we'll be in continum models.

We are familiar with the Cartesian coordinate system of x, y, z. The planes are perpendicular to each other, and all axis follow the right hand rule.

In the cylindrical coordinate system, positions are given by r, θ, z. The spherical coordinate system, which we don't need to know for this class, is r, θ, ϕ.

Vectors are denoted with arrows, and can be represented with unit vectors $\hat{i}, \hat{j}, \hat{k}$. For example, $\bar{v} = 3\hat{i}+2\hat{j}+1.5\hat{k}$. Alternatively, $\bar{A} = A_r\hat{e}_r + A_\theta\hat{e}_\theta + A_z\hat{e}_z$. The magnitutde of a vector in Cartesian is given by

$$|\bar{A}| = (A_x^2 + A_y^2 + A_z^2)^{\frac{1}{2}} \tag{15}$$

Dot (scalar) product is given by

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} = |A||B|\hat{a}\cdot\hat{b} = |A||B|\cos\theta_{AB} \tag{16}$$

where

$$\bar{A}\cdot\bar{B} = A_xB_x + A_yB_y + A_zB_z \tag{17}$$

Cross (vector) product is

$$\vec{C} = \vec{A}\times\vec{B} \neq \vec{B}\times\vec{A} \tag{18}$$

where

$$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x) \tag{19}$$

The gradient, or Del operator, is represented by ∇.

$$\nabla(\cdot) = \frac{\partial(\cdot)}{\partial x} + \frac{\partial(\cdot)}{\partial y} + \frac{\partial(\cdot)}{\partial z} \tag{20}$$

This can be used on scalars, or vectors, for example, ∇P is the gradient of pressure, while $\nabla\cdot\vec{V}$ is the gradient of the vector.

This can be used on vectors as a cross product, and we get:

$$\nabla\times\bar{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \tag{21}$$

which is also called a curl.

09/13: Hydrostatic Pressure

The TA's office hours changed, at ENGR 230A; this is a shared space hence we should be mindful of other classes. Times are Tuesdays and Thursdays at 11 AM to 12 PM. Homework 2 has been posted, and is due on Monday at 11 am (beginning of class); this will be on chapter 2, which we will cover today and on Friday.

Hydrostatics are when fluid is not moving. Pressure is due to the bombardment of individual molecules on an area,

$$P = \frac{F}{A} \tag{22}$$

An upside down triangle, ∇, on a surface, indicates being open to the atmosphere. If we take a differential cube in a fluid, the volume is $V = \partial x\partial y\partial z/2$ if we cut the cube through its diagonal. Using numerous derivations on the differential, pressure at any point in any direction is

$$p_y = p_z = p_s \tag{23}$$

that is, it is the same regardless of direction (assuming no shearing stress). This is Pascal's law.

Let p be the pressure at a pint. The force exerted by the pressure in a differential on one side is of the form

$$\left(p + \frac{\partial p}{\partial y}\frac{\partial y}{2}\right)\partial x\partial z$$

This is used to create the body force, i.e., the total force exerted by the differential. The total force is the sum of the force on all sides. In general,

$$\partial F_s = -\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right)\partial x\partial y\partial z = -\nabla p\partial x\partial y\partial z \tag{24}$$

is the pressure force.

For an incompressible fluid that is not moving, the sum of all forces is

$$-\nabla p - \gamma\hat{k} = \rho\mathbf{a}. \tag{25}$$

However, if the fluid is not moving, $\mathbf{a} = 0$, hence

$$-\nabla p - \gamma\hat{k} = 0$$

The pressure does not change as we move in the xy directions, but do in the z direction due to gravity:

$$\frac{\partial p}{\partial z} = -\gamma \tag{26}$$

The hydrostatic distribution of pressure is given by

$$p_1 = \gamma h + p_2 \tag{27}$$

where h is the height, or

$$h = \frac{p_1 - p_2}{\gamma} \tag{28}$$

where p_1 and p_2 is the pressure at two different points in a fluid.

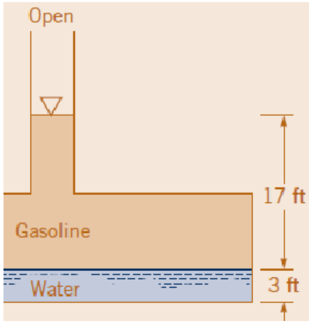

Figure 3: Pressure-Depth Relation Example

In the example 3, the top part of the oil is open to atmosphere, and hence has a pressure equivalent to that of the air, p_0. The pressure at the gas-water interface can be found with

$$p = \gamma h + p_0$$
$$p_1 = SG\gamma_{\mathrm{H_2O}} h_1 + p_0$$

If we exclude p_0, then we get gage pressure. This pressure, in terms of water, is $p_1/\gamma_{\mathrm{H_2O}}$. The pressure at the bottom of the tank is

$$p_2 = \gamma_{\mathrm{H_2O}} h_{\mathrm{H_2O}} + p_1$$

Hydraulic jacks utilize fluid pressure equivalency. For the same height, $p_1 = p_2 = F_1/A_1 = F_2/A_2$. By adjusting the area ratio, we can increase the force by a large number.

$$F_2 = F_1 \frac{A_2}{A_1} \tag{29}$$

The pressure for comprehensible fluids is non-linear, mainly for gas.

When we measure pressure, we have to mention whether it is absolute or gage pressure. However, for pressure *differences*, absolute or gage pressure is not needed.

The barometer is used to measure pressure using mercury height. This only works for atmospheric pressure. When we 'suck on a straw', we create a partial vacuum in our mouth, and the higher pressure elsewhere pushes the straw contents up.

In the mercury barometer, there is a vapor pressure in the tube; this pressure ends up being the same as:

$$p_{vapor,\mathrm{Hg}} = P_{atm} - \gamma h \tag{30}$$

The vapor presure for mercury is so low that we can neglect it.

For pressures other than atmosphere, a piezometer can be used. This uses a tube with a working fluid. This can be only used for pressures higher than atmospheric pressure. The pressure must be small. The fluid in the container must be a liquid rather than a gas.

A U-tube manometer can be used to measure larger fluid pressure differences.Fluid can be gas or liquid. In homework 2, we have to use known presure to find unknown. The pressure we are trying to measure, p_A is in the container, and

$$p_A = \gamma_2 h_{-} \gamma_1 h_1 \tag{31}$$

There are also differential u-tube manometer. The inclined manometer will not be used.

09/15: Changing Pressure

Last class talk was about U-tube manometers. These magnotometers measure gage pressure. Changes in height for gasses don't matter as much as liquids. The pressure equation is

$$p_A = \gamma_1 h_1 - \gamma_2 h_2 \tag{32}$$

where p_A is the pressure we are trying to measure; subscript 1 denotes the fluid connected to the pressure we are trying to compute, while 2 denotes the one that is open to atmosphere.

If the manometer was not open to the atmosphere, but instead to a second pressure (subscript B, or 5):

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1 \tag{33}$$

This measures gage pressure with respect to b.

An inclined manometer uses the same principle, but it is inclined so that it is easier to build.

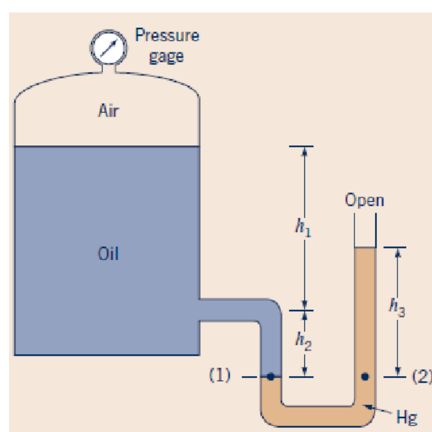

Figure 4: Simple U-Tube Manometer

Simple U-Tube Manometer

A closed tank contains compressed air and oil ($SG_{oil} = 0.9$). A U- tube manometer using mercury ($SG_{Hg} = 13.6$) is connected to the tank as shown. The column heights are $h1 = 36in$, $h2 = 6in$, and $h3 = 9in$. Determine the pressure reading 1in psi2 of the gage.

We will assume that the air has pressure. For the part of the system dealing with oil, pressure at point 1 is given by:

$$p_1 = p_{air} + \gamma_{oil}(h_1 + h_2)$$

The second equation, which is the right side for mercury:

$$p_2 = 0 + \gamma_{Hg}h_3 = P_1$$

Note that the '0' is the pressure for the open end, because we are using gage pressure relative to the atmosphere. If the left side was to be open to the atmosphere, then that pressure would also be zero.

In one of the homework problems, all points of liquid that are open to the atmosphere have the same pressure, while the one that is enclosed has a different pressure.

Fluid applies pressure on all surface. The force is perpendicular to the plane surface. The force of liquid on a tank is

$$F_R = pA \tag{34}$$

where A is the area of a flat horizontal surface.

In contrast, the force on the wall of a tank is given by

$$F_R = \bar{p}A = \gamma\frac{h}{2}A \tag{35}$$

where $\bar{p}$ is the average pressure. In this case, the resulting force is not at the sample as center of pressure.

For any incline at angle θ, the resulting force is

$$F_R = \gamma \sin\theta \int_A y \, \mathrm{d}A \tag{36}$$

where $\int_A y \, \mathrm{d}A$ is the first moment of area, and hence can be re-written as $y_c A$.

The resulting y position is

$$y_R = \frac{I_{xc}}{y_C A} + y_c \tag{37}$$

This equation is helpful in determining what are the positions where two moments due to fluids can be counteracted.

Buoyancy is the property for a body to float or sink in a fluid, such that it is only partially submerged. This is caused due to the difference in pressure forces between the bottom of the object and near the top.

The force is given by

$$F_B = \gamma_{fluid}\mathcal{V} \tag{38}$$

This formula only includes part of the body that is submerged under water. The buoyant force travels through the center of the object, and is called the center of buoyancy.

Stable equilbrium is when a displaced body returns to its originial equilibrium position. This is important if something wants to be submerged but not shifting. A fluid surface that is inclined but open to the atmosphere will still have the same pressure (i.e., equivalent to the atmosphere). Additionally, the pressure is equivalent at different equidistant points from the surface.

Homework is due on MMOFnday. For acceleration questions, check slides on chapter 2.

09/18: Bernoulli's Equation

Today's topic will be on Chapter 3. The exam will be 2 weeks from today, on October 2nd. We will have a quiz on chapter 2 next week. It will be similar to question 5 from HW 2. We'll be able to use our notes and work on it together; we'll work on it 10 minutes on our own, and then group. Practice problems for the exam can be given. We'll be given formula sheet during the exam.

Basic equation for Newton's Second Law can be applied to small fluid elements; when doing $F = ma$, we assume that it is

- steady state
- inviscid fluid
- coordinate system over streamline
- incompressible flow
- irrotational flow

Steady state means that the fluid properties will not change with respect to time. For example, flow over an airfoil at low speeds is steady state, but at high speeds the temperature changes hence is not steady state.

Inviscid fluid means no viscosity and friction. We assume no dissipation of energy in the form of heat. There will be no shear forces. The only forces will be from pressure force and gravity.

The streamline is the line tangent to the flow of the fluid. Unlike Cartesian coordinates, we will use (s, n), where s is the curved distance from the starting point, and n is the orientation. Many flows in this class will be simple. The curvature is denoted with $\mathcal{R}$. Streamlines do not cross each other. When the flow is straight, there is no curvature, but n still exists.

Incompressible flow is where $\frac{D\rho}{Dt} = 0$. There is more definition on this, but we do not have to worry about it. In general, density does not change with time, divergence of velocity is zero (no expansion/shrinking). Liquids are always incompressible. Gasses can be incompressible when less than mach 0.3, but compressible greater than that. This example is with *flow*, not necessarily with the fluid.

Irrotational flow is when the curl of the velocity is zero

$$\nabla \times \vec{V} = 0 \tag{39}$$
$$\vec{\omega} = 0 \tag{40}$$

The coordinates will be given by s. The velocity along the streamline will be $\mathbf{V} = \frac{ds}{dt}$, and acceleration is $\frac{d\mathbf{V}}{dt}$

Along the streamline,

$$\partial m a_s = \rho \partial \cancel{V} V \frac{\partial V}{\partial s} \tag{41}$$

Bernoulli's Equation

$$p + \frac{1}{2}\rho V^2 + \gamma z = c = \frac{F}{L^2} \tag{42}$$

where c is a constant, p is the pressure energy, the second term is the kinetic energy, and the third term is the potential energy. In general, this states that these energy are conserved.

The alternative equation to definition 7:

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = L \tag{43}$$

Whenever a liquid is open to the atmosphere, its pressure is zero (gage).

Bernoulli's equation can be used on two points on the stream.

09/20: Bernoulli Example Problems

Last class was about Bernoulli's equation. The width of the streamline is not considered.

Bicyle Problem

Consider the flow of air around a bicyclist moving through still air with velocity V_0. Determine the difference in the pressure between points(1) and (2).

First we need a coordinate system. Using a coordinate system that is moving with the person, then we get steady state. The velocity right in front of the biker is 0 because the biker is not moving in the relative coordinate system. By Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + \cancel{\gamma z_1} = p_2 + \cancel{\frac{1}{2} V_2^2}^{0} + \cancel{\gamma z_2}$$
$$p_1 + \frac{1}{2}\rho V_1^2 = p_2$$
$$V_1 = V_0$$
$$\therefore p_2 - p_1 = \frac{1}{2}\rho V_0^2$$

There is no change in height, so the z term is cancelled. The final equation tells us that as the air approaches the biker and becomes zero (relative to biker), the pressure increases. This is like how when we put our hand outside a moving car, we can feel the air pressure on our hand.

Football Problem

Consider the inviscid, incompressible, steady flow along the horizontal streamline A–B in front of the sphere of radius a. The fluid velocity along this streamline A–B is

$$V = V_0 \left(1 + \frac{a^3}{x^3}\right)$$

Determine the pressure variation along the streamline from point A far in front of the sphere to point B on the sphere.

The equation tells us how the air is slowing down with respect to the ball. The z component in Bernoulli's equation can be cancelled out since they are the same. Velocity at B that touches the ball is zero, so that term can be cancelled. Therefore, the equation we receive is

$$P_A + \frac{1}{2}\rho V_A^2 = P_B$$
$$\Rightarrow P_A - P_B = -\frac{1}{2}\rho V_A^2$$

The book uses a different method that is continuous, while the professor's method (above) is only at two points. The rate of change of pressure is illustrated in the form of differentials. If the professor asks us to solve it in the method of the book, he will give us the starting equation.

09/22: Bernoulli with Curvature

Exam will be next Monday (after next week). Question 1 (has two sections, we can choose to do either) from Theory chapter. Question 2 from problem 2 (8 pts). Question 3 from chapter 3 (8 pts). The extra credit (2 pts) can be received by doing the other section of question 1. Show work, indicate final answer, etc. Equation sheet posted on BlackBoard. Review problems for chapters 1 and 2 ready. Questions from HW 3 will be trimmed, and instead be in the review problems for chapter 3. We do not have to turn in the review problems; it is meant for prep. The lecture before the exam will be review.

Last class we went over an example involving pressure in front of a bicycle. One important point is to ignore the effect of height for air, since most heights are too small to have an effect. In the other example involving the soccer ball; and how the horizontal stream line does not involve any angle term; terms were substituted to derive an equation that can be used to find the change in pressure at any point in front of the ball; integrating the pressure differential allows us to find the pressure at any point.

Vertical Free Jet

This is 'Example 3' from the chapter 3 slides

We can imagine a streamline from point 1 down to point 2. Because we are on the streamline, we can assume the 5 assumptions to use Bernoulli's equation. The equation is:

$$\cancel{p_1} + \cancel{\frac{1}{2}\rho V_1^2} + \gamma z_1 = \cancel{p_2} + \frac{1}{2}\rho V_2^2 + \cancel{\gamma z_2}$$

The pressures cancel because it is open to the atmosphere. The γz_2 term is cancelled because that is the lowest point. The V_1 term goes to zero because we can assume that that point is not moving (assume the tank is large enough). The equation is now:

$$\gamma h = \frac{1}{2}\rho V_2^2$$
$$V_2 = \sqrt{2gh}$$

Notice how the final equation is equivalent to the free fall final velocity. To find the velocity even later, suppose after it has fallen an additional distance H, then:

$$V_5 = \sqrt{2g(h+H)}$$

Between points 3 and 4, z_4 can be considered 0 for being the lowest level, $V_3 = 0$ since we can assume it is stationary, $p_4 = 0$ since it is open to the atmosphere, and $p_3 = \gamma(h-l)$. As a result, the resulting Bernoulli equation is

$$\gamma(h-l) + \gamma l = \frac{1}{2}\rho V_4^2$$
$$V_4 = \sqrt{2gh}$$

Keep in mind that $\gamma = \rho \cdot g$; this is basically density, but considering gravity.

If flow occurs through changing diameter, the mass consistency requires that the velocity should also change:

$$A_1 V_1 = A_2 V_2 \tag{44}$$

A free jet, in reality, does not follow this behavior exactly, since the flow breaks apart.

The non-slip boundary condition has no effect on the pressure.

Now we will consider curved streamlines, or more specifically, forces normal to the streamline.

$$\sum \partial F_n = \partial m a_n \tag{45}$$
$$= \rho \partial \cancel{V} \frac{V^2}{\mathcal{R}} \tag{46}$$
$$\Rightarrow -\gamma \frac{\mathrm{d}z}{\mathrm{d}n} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}} \tag{47}$$
$$p + \rho \int \frac{V^2}{\mathcal{R}}\,\mathrm{d}n + \gamma z = \text{constant along streamline} \tag{48}$$

If the streamline is flat, it has an infinite radius of curvation, $\mathcal{R}$, otherwise it does. Sharper the curve, the smaller the radius of curvature.

Pressure Variation Normal to Streamline

This is example 4.a from the slides

The velocity is given by $V = (V_0/r_0)r$ on the horizontal plane. If we go along the streamline, i.e. in the revolution, the two are equivalent:
$$\frac{\partial p}{\partial n} = -\frac{\partial p}{\partial r} \ni \mathcal{R} = r.$$
The resulting equations from the derivations shown in the slides is

$$p = \frac{\rho}{2}\left(\frac{V_0}{r_0}\right)^2 + C.$$

where
$$C = p_0 - \frac{\rho}{2}V_0^2.$$

The normal vector points to the center, where as the radius vector talks about how far something is *from* the center. Example 4.b skipped.

Flow over a bump

This is example 5

We are comparing pressure differences between 2 and 1, and then 3 and 4. Between 1 and 2, the $\mathcal{R} = \infty$. We can use equation 48, which is similar to the Bernoulli equation:

$$p + p\cancel{\int \frac{V^2}{\mathcal{R}}}\,\mathrm{d}n + \gamma z = C$$
$$p_1 + \gamma z_1 = \cancel{p_2} + \gamma z_2$$
$$p_1 = \gamma(z_2 - z_1)$$
$$= \gamma h_{2-1}$$

Which is the same as hydrostatic pressure. In contrast, between points 3 and 4, we cannot cross out the integral term. The resulting equation is:

$$p_3 = \gamma h_{4-3} - \rho\int_{z_3}^{z_4} \frac{V^2}{\mathcal{R}}\,dz.$$

We use a cylinndrical coordinate system if $\mathcal{R}$ is constant.

09/25: Finishing up on Bernoulli's Equations

Example problem on slide 30/59 of chapter 3 slides will resemble one of the homework question. We need energy from the fluid to make the fluid curve; the faster the flow, the more energy is needed.

Consider flow to be confined and not exposed to the atmosphere. If we have liquid flowing through a nozzle, mass is conserved: mass going in is mass going out. The volume flow rate will also be constant:

$$Q = VA \tag{49}$$
$$\dot{m} = \rho Q \tag{50}$$

For a constant Q, halving the area will double the velocity at the outlet. For incompressible flow, $\rho_1 = \rho_2$;
$$V_1A_1 = V_2A_2 \tag{51}$$

Siphoning is where liquid can be taken out by first creating a lower pressure at the end of the tube. One of the conditions is for the outlet height to be lower than the inlet height.

Siphoning Example

See slide 43/59 of chapter 3 slides

Velocity at the inlet of the tube is equal to the outlet. Pressure at point 2 is lower than pressure at point 1, since it is higher at that point. Its gage pressure will be negative. Because pressure decreases if we increase height H, the fluid at point 2 can cavitate; when it cavitates, siphoning will not work properly. We can set or cross off some terms of the Bernoulli equation in this case:

$$\cancel{p_1} + \cancel{\frac{1}{2}\rho V_1^2} + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 == \cancel{p_3} + \frac{1}{2}\rho V_3^2 + \gamma z_3$$

The velocity at points 2 and 3 are the same. We can use points 1 and 3 to determine the outlet pressure.
Knowing that cavitation occurs at a certain pressure, we can set it to $p_2 = 0.257$ psia. The 'a' at the end of psi stands for 'absolute'. Two ways of causing cavitation is increasing the height of point 2, or decreasing the height of point 3 to increase the velocity so much that it cavitates.

We can decompose Bernoulli's equation into the summation of the static pressure, p, the dynamic pressure $\frac{1}{2}\rho V^2$, and the hydrostatic pressure γz.

These yield the total pressure. The stagnation pressure is the static and dynamic pressure combined. In slide 34/59, point 4 is static pressure (atmospheric); point (1) has dynamic since it has a flow rate. And point (2) has hydrostatic pressure. (They can have mix of all). Stagnation pressure is the pressure once flow slows down.

To measure this pressure, it is the pitot-static tube; it is used to measure airspeed. The differential in pressure can measure air speed; it measures the difference between static and stagnation pressure. This will work at all mach numbers, velocities, densities, etc.

The pitot-static tube from point 2 to 3 (slide 37) is the pitot-tube that measures stagnation pressure; points 1 to 4 is the piezometer that measures static pressure. All the edges have to be flat and smooth, because it would cause stagnation points and hence change the pressure readings.

We won't be quized on the design considerations.

Airplane Pitot-static tube example

See slide 41/59

Stagnation point is at the nose of the wing, which is point (2). Use Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + \cancel{\gamma z_1} = p_2 + \cancel{\frac{1}{2}\rho V_2^2} + \cancel{\gamma z_2}$$
$$p_1 = 1456\,\frac{\text{lb}}{\text{ft}^2}$$
$$V_1 = 200\,\text{mph}$$
$$= 293\,\text{ft/s}$$
$$p_2 - p_1 = \frac{1}{2}\rho V_1^2$$

The stagnation pressure will always be equal to or higher than the static pressure.

We can read chapter 3.7 on our own. It is where we divide Bernoulli's equation with γ, and this gives us everything in terms of length.

If the flow is irrotational, then Bernoulli's equation can be used everywhere (i.e., across streamlines or between different streamlines): $\nabla \times \vec{V} = 0$. If the flow does rotate, i.e. $\nabla \times \vec{V} \neq 0$, then the equation can only work along the streamline.

The Assassin's Teapot uses Bernoulli's equation by having two chambers whose flow can be stopped by closing the suction end. Another example of historical use of fluids is Pythoagoras Wine glass, where if the fluid height increases over a larger height, it will siphon.

09/27: Final Examples and Fluid Kinematics

Last class we learned about different kinds of pressure, confined flow, etc. Today we will go over a couple of exam and work on the examples. (Note: Examples are coming from the chapter 3 slides that have been posted on BlackBoard).

Beverage Stream Example

See slide 54/59 of chapter 3 slides

We have to use Bernoulli's equation between points 1 and 2. We can set the pressure to zero because both points are 'open' to the atmosphere. γz_2 can be set to zero because it is at the bottom point. We can assume that the fluid at point 1 is moving vertically at a speed of V_1 through a diameter of D, while the exit is at V_2 through d. Therefore, we have:

$$A_1 V_1 = A_2 V_2$$
$$V_1 = \frac{A_2}{A_1} V_2$$
$$V_1 = \frac{d^2}{D^2} V_2$$

We also modify the Bernoulli's equation:

$$[\frac{1}{2}\rho V_1^2 + \gamma h = \frac{1}{2}\rho V_2^2] \times \frac{2}{\rho}$$
$$V_1^2 + 2gh = V_2^2$$
$$(\frac{d}{D})^4 V_2^2 + 2gh = V_2^2$$
$$(1 - (\frac{d}{D})^4) V_2^2 = 2gh$$

Slide 56 completes the rest of the equation. Note that we do NOT cancel out V_1 (contrary to other examples) because we are given the diameters, and D is not *too* much larger than d.

Air hose example

See slide 57

At point 1, the velocity and γz term can be cancelled. For the others, the γz term can also be cancelled because they are the same height. At point 3, since it is exposed to the air, we can say the pressure p_3 is zero (gage).

At point 1, everything is given. At point 3, our only unknown is velocity. Therefore, we can take point 1 and point 3 to find V_3. Then, we can find the flow rate Q.
To go from volumetric flow rate, Q, to mass flow rate, $\dot{m}$, we multiply with the density:

$$\dot{m} = \rho Q$$

$\dot{Q}$ is NOT used to represent flow rate. Q is used to represent volumetric flow rate.
For the final solution of the problem, we can either relate p_2 to the point at 1, or to the point at 3, to obtain the final solution $p_2 = 2.963\,\text{kPa}$.

This is the end of chapter 3. Our exam will cover *up to* this. This Friday, we will have a review of the exam. Exam is on October 2. We can bring questions to the professor on Friday, and work on them in class. We will start on chapter 4 now. The slides for chapter 4, fluid kinematics, is already posted on BlackBoard.

Chapter 4 is about how the flow is moving, its trajectory, but are *not* too concerned about the forces.

Our particle moves along its path or stream line. At some new time $t+\partial t$, it is at a new position. If $\vec{r}$ is the position vector, we can define the velocity vector of particle A by:

$$\vec{V} = \frac{\mathrm{d}\vec{r}_A}{\mathrm{d}t} \tag{52}$$

At any point, we have a position and velocity. To look at the velocity at every point, we use Field Representation. We can also use $T(x, y, z, t)$ representation, which is both space and time.

One field representation is velocity field.

$$\vec{V} = u(x,y,z,t)\hat{i} + v(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k} = \vec{V}(x,y,z,t) \tag{53}$$

The magnitude of $\vec{V}$ is

$$V = |\vec{V}| = (u^2 + v^2 + w^2)^{\frac{1}{2}} \tag{54}$$

The two different approaches of analyzing is Eulerian or Lagrangian.

In Eulerian (spatial), we are giving properties as function of space and time. This allows us to describe the entire field better.

In Lagrangian (material), we look at how an individual particle changes as it flows. This relies on time only.

Experimental approaches can be either. For laser doppler velocimetry (LDV), lasers are pointed at a point, and the way light comes back allows us to see the speed at that point. This is an Eulerian method, since we are looking at a point only.

In Dye injection, we can inject a color, and that will follow the flow of the fluid. This is Lagrangian.

In Particle Image Velocimetry (PIV), the flow is seeded with tracer particles. Laser is used to trace the particles; it sees where the particles have moved as time changes. This is very computational, and allows reconstruction of the entire flow field. This is *both* Eulerian and Lagrangian, because we are looking at particles motion, but are also describing the entire flow field.

Flows can be 1D, 2D, or 3D. In 1D, we know only one component of velocity, such as flow in a pipe (even if the flow is not exactly uniform through the cross section of the pipe): $\vec{V}(x,t) = u\hat{i}$. For a physical water droplet that is shrinking, it is best to use spherical coordinate system, not cartesian. When we use spherical coordinates, only r term changes, which is only 1 dimensional. In essence, even if materials are 3D, they can be still 1D.

In 2D flow, there is two distinct flows: $\vec{V}(x,y,t) = u\hat{i} + v\hat{j}$.

Streamlines cannot converge. In a flow field, it may look like they are converging, but they can only get closer and closer, but will not.

3D flow is a complex flow. This would be like in a turbine. Although the flow is moving through the turbine, there are so many parts that are moving in a turbine that we have to use 3D. Example would computational fluid dynamics. $\vec{V}(x,y,z,t) = u\hat{i} + v\hat{j} + w\hat{k}$.

Flow can be steady or unsteady. If velocity changes at a given position, it is unsteady.

Laminar flow is smooth flow. It comes from the word *laminae*, meaning layers. There is very little mixing between layers. Transitional flow is where it is mostly smooth, but there is some mixing between the layers. In turbulent flow, it is very violent and mixing.

The turbulence of a flow can be characterized by the Reynolds number. This will be the most important non-dimensional number in this class.

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{VD}{v} = \frac{\text{Inertial effect}}{\text{viscous effect}} \tag{55}$$

where ρ is density, V is velocity, D is characteristic length (or diameter, this depends on the thing we are looking at), μ is the dynamic viscosity, and $v = \frac{\mu}{\rho}$.

To increase Reynold's number, we can increase velocity, dcrease viscosity, etc. It is very difficult to predict weather is because air is very unpredictable. Honey is very predictable because it has very low Reynold's number. Flow can be laminar as it starts, and then become turbulent after.

Next topic would be about streamlines, streaklines, and pathlines, which will be covered in next class (Friday's class is review).

09/29: Exam Prep Review

Today's class is a review. The first question from the review sheet is on the dimensions of the equation

$$Q = \frac{\pi R^4 \Delta p}{8\mu l}$$

We can choose one from the 3 systems: MLT, FLT, BLT. Only the first two can work. We chose FLT.

$$\begin{aligned}[Q] &= \frac{L^4 \frac{F}{L^2}}{\frac{FT}{L^2} L} \\ &= \frac{FL^2}{\frac{FT}{L}} \\ &= \frac{L^3}{T}\end{aligned}$$

An equation is homogenous if all the terms in it have the same unit.

We are expected to know how to relate dimensions to given quantities. If we don't know the dimensions, we can always use their original formula to get the dimensions.

When taking the moment of inertia, we always take the horizontal to be b, and the vertical to be a. For the quiz we had, we first had to find the centroid of the vertical component of the support, OA. This was 3 meters below the surface, 4 meters in length, and 3 meters in depth/width. Using the equation in the equation sheet, the b is the horizontal part, i.e. $b = 3\,\text{m}$. The centroid, in this case, is the geometric center, $y_c = 2\,\text{m}$; if we include the distance from surface, it becomes $h_c = 3\,\text{m} + y_c$. Then, we can use the equation $y_R = \frac{I_{xC}}{y_C A} + y_C$. This is where the force *acts*, but the force itself is $F_R = \gamma h_C A$.

For an angled surface, the equations will hold true, but with modifications. The y term will be angled, since it is geometric to the surface, but the h term will remain vertical. Trigonometric relationship will form between h and y.

For chapter 3 example 2 (from review questions), wind starts at 40 mph, and increases to 60 mph as it goes over the roof. Here, use Bernoulli's equation. We ignore change in height, since it's air:

$$p_0 + \frac{1}{2}\rho V_0^2 + \cancel{\gamma z_0} = p_1 + \frac{1}{2}\rho V_1^2 + \cancel{\gamma z_1}$$

$$\begin{aligned}p_0 &= 14.7\,\frac{\text{lb}}{\text{in}^2} \cdot \frac{(12\,\text{in})^2}{(1\,\text{ft})^2} \\ &= 2116.8\,\frac{\text{lb}}{\text{ft}^2} \\ V_0 &= 40\,\frac{\text{mi}}{\text{h}} \to [\text{ft/s}] \\ &= 58.\bar{6}\,\frac{\text{ft}}{\text{s}} \\ V_1 &= 88\,\frac{\text{ft}}{\text{s}} \\ \rho_{\text{air}} &= 2.38 \times 10^{-3}\,\frac{\text{slug}}{\text{ft}^3} \\ p_1 &= 2116.8\,\frac{\text{lb}}{\text{ft}^2} - \frac{1}{2}(2.38 \times 10^{-3}\,\frac{\text{slug}}{\text{ft}^3})[(58.7\,\frac{\text{ft}}{\text{s}})^2 - (88 \\ &= 2111.687\,\frac{\text{lb}}{\text{ft}^2} \\ &= 14.66\,\text{psi}\end{aligned}$$

The roof here will be pulled up slightly. We'll be given default values such as air pressure and water density. We'll also be given conversion factors.

At low velocities, air can be treated as incompressible.

For question 4 from chapter 2 review, we calculate pressure using the manometer relationship. Initially, there was positive pressure in the air compartment, but after the hole, the three liquids will rise on the left side, and the mercury on the right will decrease. The equation is given by:

$$P_a + \gamma_b(0.1) + \gamma_w(0.1) + \gamma_m(0.1) - \gamma_m(0.3) = 0$$

The individual heights of benzene and water will remain unchanged; mercury will change. But the volume displaced on the left should be same as the one on the right for mercury. So we get another equation relating that:

$$\begin{aligned}\Delta h^* D^2 &= \Delta h d^2 \\ \Delta h^* &= \Delta h(\frac{d}{D})^2 \\ &= \frac{\Delta h}{100}\end{aligned}$$

The change in height on the left is very small, so we can neglect h^*.

10/03: Lines and Acceleration

We should not talk about the exam. Laminar flow can sometimes look 'frozen'. As velocity is increased, flow goes from laminar to transitional to turbulent. For an airfoil, which is much more aerodynamic, needs higher Reynolds number for turbulence.

A streamline that is tangential to the flow at every point. For a steady state, the streamlines are fixed in space.

(Note that chapter 4 slides have been posted on BlackBoard).

Velocity Field and Streamlines

See slide 17/69 of chapter 4 slides

For the first question, we write V in its component form. Since this is a 2D flow, there is no w component. We replace the u and v with their respective i and j components. Pythogoras theorem can be used to obtain the magnitude.
For the sketch of the velocity field, we draw it in terms of $\theta = \tan^{-1}\left(\frac{v}{u}\right)$. With this, we have the angle, and the magnitude from answer (1), so these two can be used to draw the velocity field.
To obtain the streamline equation, we first consider the general equation,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v}{u}.$$

and substitute the terms u and v. The integration of this gives us the equation

$$xy = C.$$

Note that when doing integrals, the constant can change but yet be called the same thing; e.g., we may use C for both C and $\ln C$.

A streakline is the collection of all particles that passed through a common point. This is not a mathematical tool, but more of a visual tool. The video example shown in class with the blue dye is a streak line, since all those particles came from the same point.

The pathline is the Lagrangian approach where we track down a single particle and painting the line that particle took.

Streakline is a collection of a particle going through one point. Pathline is a single particle going through many places.

All particles going through the same place makes streamlines, streaklines, and pathlines be the same.

The chain rule on the velocity is used to determine the acceleration. We notice that the derivative of position with time is just the velocity. The acceleration will also end up being a vector. The $\frac{dV}{dt}$ term is the change of the velocity of the field, at that point. The other terms represent the change as we move from that point.

The material derivative is given by the capitalized D:

$$\frac{D(\cdot)}{Dt} \equiv \frac{\partial()}{\partial t} + u\frac{\partial()}{\partial x} + v\frac{\partial()}{\partial y} + w\frac{\partial()}{\partial z} \tag{56}$$

$$\equiv \frac{\partial()}{\partial t} + (V \cdot \nabla)() \tag{57}$$

Acceleraation along a streamline

See slide 31/69 of chapter 4 slides

We are just interested in the x direction. We want the acceleration along the x axis streamline. Velocity at A exists, but at B it is zero; we should therefore except the velocity to decrease.
There is only one velocity component, x, therefore we can simplify:

$$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i}.$$

Here, we assume that this is steady. Velocity does not change with respect to time; it changes only respect to position. So we only have

$$a_x = u\frac{\partial u}{\partial x}.$$

By substituting the equation for V, we can get the final acceleration.

If there is unsteady effect, then the properties at the locations can change with time. For example, the velocity at one point can be increasing or decreasing; this would make the $\frac{\partial V}{\partial t}$ be *non*-zero. Additionally, the rest of the terms are also non-zero if the velocity also changes as we move in space.

10/06: Exam Results Review

Average for class was slightly below 11. Exam is lower than quiz is lower than homework grades. 1 person finished in 30 minutes. Everyone finished after 50 minutes. Most people chose question 1a for full points. Most people got this question right. Question 1b was bimodal: either very good or very bad. But those who did 1b for full points, most got right. For question 2, the average was 60%. For question 3, the average was 25%.

The BlackBoard grade has multiple types of grades. Our early grade is the expected grade based on the content we have done. The average early grade for this class is 68%. However, we should not take it as a dread of the class. We will get extra credit opportunities, and the project will boost our grades. The project maybe 15% of the grade.

The answers to the exam will not be posted on BlackBoard. For question 1a, the answer was 1, i.e., dimensionless. In questien 1b, it didn't matter if we used MLT or FLT, since we'd still get L^2/T.

For problem 2, we needed pressure at point a. We would have to start from the water, go to liquid A, then to the other side, and then to B, and finally to the atmosphere. Since we were using gage pressure, atmospheric pressure was zero. However, if we did absolute pressure, we would still get grades. Most people got part 2a correct. In 2b, the level of water in liquid B tank will decrease. For this, we had to do conservation of volume.

$$\begin{aligned}\Delta h^* A_1 &= \Delta h A_2 \\ d_1 &= 3d_2 \\ \Delta h^* &= \left(\frac{d_2}{d_1}\right)^2 \Delta h \\ &= \frac{\Delta h}{9}\end{aligned}$$

We had to give justification, about how much will decrease. When the pressure in a decreases, the level of water in liquid A on the right side will decrease by Δh, and the one on the left will increase by Δh, therefore the total movement is actually $2\Delta h$. In Engineering, if the difference is two orders of magnitude, it can be ignored; however, in this problem, the change in height in the big tank was $\Delta h^* < 100\Delta h$, therefore it could not be ignored.

For problem 3, we got the hint what the cavitation pressure was; we were supposed to use the left column. In the right section, water will move through the hose. By comparing the barometer's top and bottom, we can find cavitation pressure. Since it is gage pressure, it would be negative. After that, we could relate the tank's surface to point at top of the siphoning tube, and solve for the velocity there. We have a confined flow between the top point and the end of the tube o the right side, therefore can use conservation of volume. Using the velocity, and the fact that

$$V_3 = \sqrt{2gh} \tag{58}$$

to obtain the height. For 3b, we would use confined flow, and also use the free fall equation 58. Two methods of creating cavitation is to increase the height. The other method is for dynamic pressure.

We'll do system and control volumes later. Homework 4 will be released (likely) on Monday, and due the Monday of the following week.

10/09: Control Volumes and Systems

The project will now be project proposal followed by the actual project. We will get extra credit if we talk about things that have not been covered. Office hours will not be on Wednesday, but Tuesday, for this week.

Last class, we talked about the acceleration vector, by taking the material derivative of the velocity with respect to time. Since acceleration is a vector, it has scalar components in the three directions. This type of derivative is called the material derivative:

$$\frac{D()}{Dt} \equiv \frac{\partial()}{\partial t} + (\vec{V} \cdot \nabla)() \tag{59}$$

The material derivative can be used on anything.

We went over a tennis ball example, where the acceleration was looked at in the x direction.

The acceleration can be converted into different coordinate system. One of the homework is to use cylindrical coordinates.

An example of convective effect is like a water heater, where the process is constant, but it is different at different place. Another example is a wind tunnel, where it is steady state, but the velocities and accelerations at different places are different.

Another example of the material derivative is temperature in space. The convective derivative component is:

$$\nabla T = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} \tag{60}$$

For a wind tunnel, we can assume 1D flow since it is cylindrical. Additionally, the velocities change with respect to the radius of the tunnel at different areas. For this,

$$\vec{a} = u\frac{\partial u}{\partial x} \tag{61}$$

which is for wind tunnel on slide 37 in chapter 4 slides.

Acceleration from Given Field Vector

See slide 38 of 69 of chapter 4 slides

The definition for acceleration is

$$\vec{a}(t) = \cancel{\frac{\partial V}{\partial t}} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} w\cancel{\frac{\partial V}{\partial z}}$$
$$a_x(t) = u\frac{\partial u}{\partial x} + v\cancel{\frac{\partial u}{\partial y}}$$
$$a_y(t) = u\cancel{\frac{\partial v}{\partial x}} + v\frac{\partial v}{\partial y}$$
$$u = -\frac{V_0 x}{l}$$
$$v = \frac{V_0 y}{l}$$

We can cancel the terms that do not have change (derivative). Additionally, in the a_x and a_y terms, we can cross out the partial derivatives that are not changing.

$$\frac{\partial u}{\partial x} = -\frac{V_0}{l}$$
$$\frac{\partial v}{\partial y} = \frac{V_0}{l}$$
$$\Rightarrow a_x(t) = \left(-\frac{V_0 x}{l}\right)\left(-\frac{V_0}{l}\right)$$
$$\Rightarrow a_y(t) = \left(\frac{V_0 y}{l}\right)\left(\frac{V_0}{l}\right)$$
$$\therefore \vec{a} = \frac{V_0^2 x}{l^2}\hat{i} + \frac{V_0^2 y}{l^2}\hat{j}$$

We'll have a couple of homework problem related to drawing streamlines and fields.

A system is a fixed amount of matter. Matter that goes in, goes out. In practical situations, it is hard to do.

Control volume is a selected region of space that we do our analysis in. This volume can flow. This is easier to do.

The thrust of a rocket is by the propellent. If we did mass to analyze, the problem would be that the rocket is releasing the mass. We can, however, use control volume to actually see the amount of mass leaving, and that way measure the amount of thrust.

The control volume can be fixed, like in a pipe; it can be moving, like the engine of an airplane (since the airplane moves, the engine moves, and hence the volume moves along); it can deform and move, like a balloon ejecting air.

Laws of fluids apply to system approach directly. This is not true for control volume.

Some basic laws for system:

- Conservation of mass
$$\frac{\mathrm{d}m}{\mathrm{d}t}|_{sys} = 0 \tag{62}$$
- Conservation of linear momentum
$$\vec{F} = \frac{\mathrm{d}m\vec{V}}{\mathrm{d}t}|_{sys} \tag{63}$$
where the $m\vec{V}$ is the linear momentum.
- Conservation of angular momentum
$$\vec{T} = \frac{\mathrm{d}(\vec{r}\times\vec{V})\cdot m}{\mathrm{d}t}|_{sys} \tag{64}$$
- Conservation of energy
$$\dot{Q} - \dot{W} = \frac{\mathrm{d}E}{\mathrm{d}t}|_{sys} \tag{65}$$

The Reynolds Transport Theorem is used to see effects of fluid with an object. As an example, measuring the amount stuff coming out of a rocket engine to determine the thrust.

We'll talk about extensive and intensive properties. Note that intensive poperties are always given by
$$b = \frac{B}{m} \tag{66}$$
where B is the extensive property.

Mass per unit mass is m/m, which is 1. Temperature is also weird, because the intensive property for temperature is T/m.

10/11: Control Volumes and Systems

There will be a 5 pt extra credit quiz on Friday. There's delta credit on this as well. It will be similar to previous quizzes and homework.

The Reynolds Transport Theorem is a means of connecting the different systems. Extensive properties are denoted by capital B, while intensive is b, and the relation is $B = mb$. The extensive property of the system B_{sys} can be determined by summing up all the individual properties.

$$B_{sys} = \lim_{\delta V\!\!\!/\to 0}\sum_i b_i(\rho_i \delta V\!\!\!/_i) = \int_{sys} \rho b \,\mathrm{d}V\!\!\!/ \tag{67}$$

We take as many elements as possible, which causes the size of the volume to be near zero. Note that $\rho\delta V\!\!\!/$ is the differential mass. Lower case b is unitless if mass, otherwise it can have different units.

Many laws are time dependent. If we change the equation to the derivative, we get

$$\frac{\mathrm{d}B_{sys}}{\mathrm{d}t} = \frac{\mathrm{d}\int_{sys} \rho b \,\mathrm{d}\mathcal{V}}{\mathrm{d}t} \tag{68}$$

We can do the same with control volume.

Suppose an example where there is pressurized fluid in a tank that is released to let the fluid exit. The control volume (tank) remains same, but the mass decreases (released). Note how:

$$\frac{\mathrm{d}\int_{sys} \rho \,\mathrm{d}\mathcal{V}}{\mathrm{d}t} = 0 \tag{69}$$

$$\frac{\mathrm{d}\int_{CV} \rho \,\mathrm{d}\mathcal{V}}{\mathrm{d}t} < 0 \tag{70}$$

The volumetric rate of change is $\frac{\mathcal{V}}{t} = AV$. If we are concerned about the mass flow rate, it is equivalent to

$$\dot{m} = \rho AV \tag{71}$$

For the derivation of the Reynolds Transport Theorem, consider the control volume. After some δt, the system boundary shifts (mass), while the control volume remains the same. However, although mass has exited the control volume, mass has also entered. Refer to slide 54 of 69 of chapter 4 slides.

We can assume that the particles moved a distance of $\delta l_1 = V_1 \delta t$. In flow is where mass enters, and outflow is where it exits. At the starting time, the $B_{sys}(t) = B_{cv}(t)$. After some time δt, the system is the control volume plus the outflow minus the inflow:

$$B_{sys}(t+\delta t) = B_{cv}(t+\delta t) - B_I(t+\delta t) + B_{II}(t+\delta t) \tag{72}$$

We can take the derivative, and also do some substitutions. Refer to equations in slides 56 and 57. For the inflow and outflow, we use the volumetric rate of change and substitute it into the green and red equations.

If the following conditions are met:

1. Control volume is fixed (in contrast to how some control volume can move)
2. One inlet and one outlet (in the future, we can extend this to multiple)
3. Velocity is normal to the inflow and outflow boundaries (in order for *this* equation to hold true
4. The properties of b are unchanged

then the Reynolds Transport Theorem is

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \rho_2 b_2 A_2 V_2 - \rho_1 b_1 A_1 V_1 \tag{73}$$

In the fire extinguisher example, the amount of mass does not change, and there is no inlet, therefore these terms are zero:

$$\begin{aligned} \cancel{\frac{DB_{sys}}{Dt}} &= \frac{\partial B_{cv}}{\partial t} + \rho_2 b_2 A_2 V_2 - \cancel{\rho_1 b_1 A_1 V_1} \\ \frac{\partial B_{cv}}{\partial t} &= -\rho_2 A_2 V_2 \\ &= -\dot{m} \end{aligned}$$

In general, the material derivative of the system and the control volume's derivative remain the same while the $\rho_2 b_2 A_2 V_2 - \rho_1 b_1 A_1 V_1$ are modified. So long as the control volume is fixed in space, we can consider flows that are not normal to surface, multiple inlet and outlet, and changing properties. Keep in mind that the $\rho_2 b_2 A_2 V_2 - \rho_1 b_1 A_1 V_1$ terms change as a result.

We are evaluating the net change per time when we say $\dot{B}$. We will have to take the integral at the inlet and outlet.

$$\delta B = \rho b \delta \mathcal{V} \tag{74}$$

Note that the volume is equal to product of velocity and area and time:

$$\delta B = \rho b V \delta A \delta t \tag{75}$$

and if the velocity is not perpendicular to the surface, then

$$\delta B = \rho b (V \cos(\theta) \delta t) \delta A \tag{76}$$

For $\delta \dot{B}$, we take derivative of above.

For the inflow, we have to put a negative sign in front of it. If we combine the integrals for $\dot{B}_{out}$ and $\dot{B}_i n$, we get a single integral. The 73 now becomes

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b \,\mathrm{d}\mathcal{V} + \int_{cs} \rho b \vec{V} \cdot \hat{n} \,\mathrm{d}A \tag{77}$$

This is hard to do generally, so we'll do simple cases only.

If we look at the material derivative, the $\frac{\partial ()}{\partial t}$ is like the control volume, while the $\vec{V} \cdot \nabla$ term is the surrounding region.

Quiz will happen in the last 20 minutes of next class.

10/13: Homework and Quiz Questions

The project proposal can also be presentation and video. We should just let the professor know what we are doing with the proposal. Additionally, it is due at 12:05 on Sunday. We get extra credit for working on something not covered in class. We do not have to go super in-depth with the mathematics. There is no specific format for the project nor the references in the proposal. In the actual project, we may not need to actually recreate the phenomena. If the phenomena is not readily present, we can recreate it. For something like a hurricane, we can use past online examples.

For the first question in the homework, we have to use cylindrical coordinates. θ can be both in degrees and radians. The answer for (c) is dependent on the answer for (a) and (b).

For number 3 in the homework, the flow is steady because there is no time variable.

For number 5, the local component is $\frac{\partial \vec{V}}{\partial t}$. The total acceleration is the combination of local and convective components.

For number 4 has to have 3 equations for the different acceleration components. Instead of x and y, we'll have θ and r. The equations that follow question 4 will be useful in manipulating the equations. In cylindrical questions, effects on r also effect θ. We should search up online on it, and identify how to handle the additional terms that we get when we attempt to determine the acceleration.

For question 3, the time of zero is at $x = 1$. If the velocity is not constant, we can use an integral form. $v = \frac{\mathrm{d}s}{\mathrm{d}t}$ which can be converted to the integral.

Quizzes were passed out.

10/16: Using Reynolds Transport Theorem

We will not cover the results of the quiz as of now, but can talk to the professor after. From last lecture, we found the equation for Reynolds Transport Theorem. When we are given the general form of the equation, we need to identify the number of outlets, and modify the equation accordingly. For each inlet, we need a $p_i b_i A_i V_i$ term. The control volume can move.

Consider the Moving Control Volumes example on slide 64/69. The nozzle's flow eventually enters the control volume and exits. We have to take the perspective of the control volume. We have to take the relative velocity: $100 - 20 = 80\,\mathrm{ft/s}$. In other cases, the velocity of the CV may not be on the same direction as the flow, so in that case we use summation of vectors.

The equation for this case is

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} pb\,\mathrm{d}V\!\!\!/ + \int_{cs} pb\vec{W}\cdot\hat{n}\,\mathrm{d}A \tag{78}$$

We should select the control volume such that it makes our math easier. We should try make the control surface perpendicular to the flow. The point we look at should also be on the boundary of the control volume ideally.

This was the conclusion of chapter 4. Chapter 5 is the utilization of these on what we learned. Chapter 5 slides are on BlackBoard.

We will continue using the Reynolds Transport Theorem here. The intensive property for mass is 1, and for linear momentum it is the velocity.

The conservation of angular momentum will not be prioritized, but if we need it, it is available in the book. Conservation of angular momentum does not imply irrotational flow.

If someone says they can create mass out of nothing, or have a perpetual machine, they are lying.

If the $\frac{DB_{sys}}{Dt}$ term is zero, it doesn't mean that the CV and CS terms are zero. Reynolds Transport Theorem and Conservation of Mass are used interchangeably. This can work with two different fluids as well. It's best to use the equation when the system (mass) coincides with the control volume.

If the flow is steady, then ρ is constant. The mass flow rate is found by $\dot{m} = \rho Q = \rho A V$.

If the mass flow rate is

$$\dot{m} = \rho A\bar{V} = \int_A \rho\vec{V}\cdot\hat{n}\,\mathrm{d}A \tag{79}$$

then we can get the average velocity to be

$$\bar{V} = \frac{\int_A \rho\vec{V}\cdot\hat{n}\,\mathrm{d}A}{\rho A} \tag{80}$$

Ventilation Example

See slide 9/91 of chapter 5 slides

We have to calculate the volume, followed by the volumetric flow rate that we need:

$$Q = \frac{V\!\!\!/}{t} = \frac{360\,\mathrm{ft}^3}{3\,\mathrm{min}} = 120\,\mathrm{ft}^3/\mathrm{min}$$

Next, we compute the velocities. The control volume is steady, hence it is zero. We get:

$$\int_{cs}\rho\vec{V}\cdot\hat{n}\,\mathrm{d}A = 0$$
$$\Rightarrow \Sigma\dot{m}_{out} - \Sigma\dot{m}_{in} = [\rho AV]_{in}^{out}$$
$$[AV]_{in}^{out} = 120\,\frac{\mathrm{ft}^3}{\mathrm{min}}$$

The AV product going in should be same as going out (which is 120), and using that we can calculate the velocities.

Example 2: Airflow

See slide 11/91

We have different pressures on either ends. Since the control volume is constant, the $\int_{cv} \rho \, \mathrm{d}V\!\!\!/$ term is zero. We can break down the CS integral term into

$$\int_{cs} \rho \vec{V} \cdot \hat{n} \, \mathrm{d}A = \dot{m}_2 - \dot{m}_1$$
$$\rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2$$
$$\bar{V}_1 = \frac{\rho_2}{\rho_1} \bar{V}_2$$
$$\rho_1 = \frac{P_1}{R_1 T_1}$$
$$\rho_2 = \frac{P_2}{R_2 T_2}$$
$$\Rightarrow \bar{V}_1 = \frac{P_2 \cancel{R_1} T_1 \bar{V}_2}{P_1 \cancel{R_2} T_2}$$
$$= 219 \, \mathrm{ft/s}$$

Note that we MUST use ABSOLUTE pressure and RANKINE temperature.

10/18: Using Reynolds Transport Theorem

Homework 4 was handed out. Last class, examples of the worker air exchange and steady flow across a pipe was done.

Example 3: straight pipe

See example 3 on page 13/91 of chapter 5 slides

In this example, there's viscuous effects, and therefore at the outflow there is zero velocity near the wall, and high velocities near the center. The flow coming in is uniform. Normally, this would be a 1D flow, but there is a parabolic distribution of flow in the pipe. The equation for velocity at the outlet is

$$u_2 = u_{max} \left[1 - \left(\frac{r}{R} \right)^2 \right] \tag{81}$$

While for the inlet, it is constant U. We assume a steady flow. We have 1 inlet and 1 outlet. Therefore, the equation

$$\int_{cs} \rho \vec{V} \cdot \hat{n} \, \mathrm{d}A = 0$$

as

$$\rho A V \Big|_{out} - \rho A V \Big|_{in} = 0$$

At the inlet section, the integral is

$$\int_1 \rho \vec{V} \cdot \hat{n} \, \mathrm{d}A = -\rho_1 A_1 U$$

and at the outlet it is

$$\int_2 \rho \vec{V} \cdot \hat{n} \, \mathrm{d}A = \rho_2 \int_0^R u_2 2\pi r \, \mathrm{d}r$$

This was derived by the area of a circle:

$$\int_0^{2\pi} \int_0^R r \, dr \, d\theta = \frac{r^2}{2} \cdot \theta \Big|_0^R \Big|_0^{2\pi}$$

or (which is actually used in this case, but the top one is one we should know)

$$\int_0^R 2\pi r \, dr = 2\pi \frac{r^2}{2} \Big|_0^R$$

We cancel out the density between the inlet and the outlet because they remain unchanged. When we substitute the value for u_2, at that step we also distribute the r term, so that the integral can be done separately for both terms.

$$\pi u_{max} \int_0^R \left[1 - \left(\frac{r}{R} \right)^2 \right] r \, dr = 2\pi u_{max} \left[\int_0^R r \, \mathrm{d}r - \int_0^R \frac{r^3}{R^2} \, \mathrm{d}r \right]$$
$$= 2\pi u_{max} \left[\frac{r^2}{2} - \frac{r^4}{4R^2} \right]_0^R$$
$$= \frac{\pi u_{max} R^2}{2}$$
$$\Rightarrow u = 2U$$

Additionally, the average velocity at both sections is the same because the areas are the same:

$$\bar{V}_1 = U \bar{V}_2$$

This is a very common problem that we will see often; in particular, the integration.

Example 4: poor workers in a trench

See slide 16/91

In this example, CO2 is going in, and air is going out. Our starting equation is

$$\frac{\partial}{\partial t}\int_{cv}\rho\,\mathrm{d}\forall+\int_{cs}\rho\vec{V}\cdot\hat{n}\,\mathrm{d}A=0$$

Since they have different densities, the mass in this system will change. Hence, this is an unsteady problem. The mass will be increasing all the time.
In this problem, it is easier to work with two separate species: one for each gas. This is shown in the long equation on slide 17/91. We can break down the equation of the two species into their own:
We assume the gas sinks down, so it has its own volume. The height of CO2 is h. We can talk about volumetric flow rate as $Q = AV$, while mass flow rate as $\dot{m} = \rho AV$. Therefore, they are related as $\dot{m} = \rho Q$. The method in the slides is the more general approach to doing these types of problems.

Generally, we keep a positive sign convention for outflow, and negative for in flow. If flow is incompressible, it is recommended to use the $Q = AV$ equation for simplicity. Also, if we use the mass equation, we should use $\dot{m} = \rho A\bar{V}$, where the $\bar{\ }$ indicates 'average'.

Now we will consider moving control volumes, though still nondeforming.

Example airplane

See slide 22/91

Here, there's three in/out-lets. We don't have to consider the area of pipe to the fluid, just the mass flow rate. The velocity of the flow with respect to the engine is $W = V - V_{CV}$. The $V_{CV} = 971\,\mathrm{km/h}$. Meanwhile, V_1 is 0 because the air is stationary. Also, since the plane is moving in the negative direction:

$$W_1 = V_1 - V_{CV} = 0 - (-971) = 971\,\mathrm{km/h}$$

Similarly, for the outlet,

$$W_2 = V_2 - V_{CV} = 1050 - (-971) = 2021\,\mathrm{km/h}$$

We are considering the air to be in steady state. Therefore, the equation is

$$\int_{cs}\rho\vec{W}\cdot\hat{n}\,\mathrm{d}A=0$$

The mass flow rate of the equation is:

$$-\dot{m}_{fuel} - \rho_1 A_1 W_1 + \rho_2 A_2 W_2 = 0$$

10/20: Deforming Control Volumes

The next exam is scheduled for November 1st. However, the professor is planning on moving it to November 6th, so that we can have quiz or homework on the conservation of mass chapter. The purpose of the project is so that we can have a 'take-away'. The quiz will be sometime next week. We'll have a conservation of mass/momentum quiz.

Previously, we talked about moving, non-deforming control volumes. By using the balance of the mass flow rate, the mass flow in was solved for.

Although the mass flow rate of fuel was high, the flow rate of the air is much much higher:

$$\dot{m} = \rho AV = 1.75\times 0.8\times 971\times 10^3 = 582600\,\mathrm{kg/h}$$

This airplane example was an example of a black box, where we don't really know what's going on inside, but we can know the process at the surfaces.

Sprinkler rotating example

See slide 24/91 in chapter 5 slides

We are asked to find the speed of water leaving the nozzle. Generally, W denotes relative velocity, while V denotes absolute. To solve this problem, we can assume that the sprinkler is filled with water, so the overall mass should not change. We have one in flow and two outflow. Therefore, our equation is

$$\rho_2 A_2 W_2 + \rho_3 A_3 W_3 - \rho_1 A_1 W_1 = 0$$

where the subscript 1 is the inflow, and the rest are sprinkler outlets. Since density is the same, we can remove the ρ term from the equation. Additionally, since the flow in is equal to Q, and the areas at the outlet are the same $W_2 = W_3$, then

$$\begin{aligned} Q_2 + Q_3 - Q_1 &= 0 \\ Q_2 &= Q_3 \\ \therefore 2Q_2 &= Q_1 \\ \Rightarrow 2A_2W_2 &= Q_1 \\ W_2 &= \frac{Q_1}{2A_2} \end{aligned}$$

We can assume $Q_2 = Q_3$ because of symmetry. If we were asked for the actual velocity of the fluid instead of the relative, then we can use the equation

$$W = V - V_{cv}$$

Also, even though the RPM is increasing, the *relative* velocity will remain unchanged.

When the control volume deforms, we add an extra W_{CS} term.

$$\frac{DM_{sys}}{DT} = \frac{\partial}{\partial t}\int_{cv} \rho \, \mathrm{d}V\!\!\!/ + \int_{cs} \rho \vec{W}_{cs} \cdot \hat{n} \, \mathrm{d}A = 0 \quad (82)$$

Where CS subscript is the control surface

Syringe example

Page 28 of 91

Q_2 is exiting. There is some leakage Q_{leak}. We want to find the velocity by which the control surface, on the left side, moves to the right. The first question to answer in this problem is whether

$$\frac{\partial}{\partial t}\int_{cv} \rho \, \mathrm{d}V\!\!\!/$$

is zero. The answer is no. This is because mass in the volume is changing. Therefore, our equation is now

$$\frac{\partial}{\partial t}\int_{cv} \rho \, \mathrm{d}V\!\!\!/ + \dot{m}_2 + \rho Q_{leak} = 0$$

We will be replacing the first term with the change in mass equation. The change in mass will be dependent on the change in length $\updownarrow$ of the syringe:

$$\int_{cv} \rho \, \mathrm{d}V\!\!\!/ = \rho A_1 \frac{\partial \updownarrow}{\partial t}$$

We'll be ignoring the needle section to make our lives easier. We are not given the area of the needle either; just the Q_2. Also, $Q_{leak} = 0.1Q_2$, hence our equation comes out to be

$$\rho A_1 \frac{\partial l}{\partial t} + \rho Q_2 + 0.1\rho Q_2 = 0$$

We can cancel out the densities, since it does not change.

For this class, we'll consider liquids to be incompressible. Additionally, in this class we are not really concerned about the shape of the control volume, but rather, the inlets and exits, and areas.

Quiz was passed out.

10/23: Conservation of Linear Momentum

Quiz was turned in. For question 6 of the homework, we have to do integration from 0 to 10 ft, and then a second integration from 10 ft to 20 ft. Note that when parameters are constant, such as ρ, we can pull it out of the integral.

The extra credit from the homework 5 is going to apply over the final grade. For question 7, we have to carry out derivation of Reynold's Transport Theorem.

The conservation of linear momentum is given as

$$\vec{F} = \left.\frac{\mathrm{d}(m\vec{V})}{\mathrm{d}t}\right|_{sys} \quad (83)$$

Linear momentum is how rockets propel. We use the time derivative of mass:

$$\frac{D}{Dt}\int_{sys} \vec{V}\rho \, \mathrm{d}V\!\!\!/ = \Sigma \vec{F}_{sys} \quad (84)$$

to have an equation for determining forces. If we see forces, we should use this above equation (conservation of linear momentum). If we don't see forces, we can use conservation of mass.

For the control volume, we have linear momentum exiting and entering at the surfaces.

$$\frac{\partial}{\partial t}\int_{cv} \vec{V}\rho \, \mathrm{d}V\!\!\!/ + \int_{cs} \vec{V}\rho\vec{V} \cdot \hat{n} \, \mathrm{d}A = \Sigma \vec{F} \quad (85)$$

We have to draw a free body diagram. We will have a body force and surface force. Body forces are things like weight of liquid in the control volume. Surface force includes shearing forces and pressure forces.

Table 1: Different Forces in CV

Body	Surface
Weight	Pressure
Magnetic	Shear
Electrical	

We will have to break down the components of the forces. Note that pressure inserts a force. If the pressure is not uniform, then we can integrate:

$$F_P = \int P \, \mathrm{d}A \tag{86}$$

We will not see a lot of shear forces.

In equation 85, we should break the first two $\vec{V}$ terms into u, v, w, but keep third $\vec{V}$ term as a vector. This is because $\vec{V} \cdot \hat{n}$ returns a scalar.

Change in flow direction

See slide 34/91 of chapter 5 slides

The momentum pushes the vane, so it needs a countering force to keep it stationary. The first step to solving is to (1) define control volume. When doing this, keep velocity normal to the surface. Step 2 is to break down the velocity component. In this case, $\vec{V} = u\hat{i} + w\hat{j}$. Next, (3) draw FBD to see the forces. The forces F_A can be written in any direction, so long as we show that these are the unknown. We are neglecting gravity and friction in this case. Since atmospheric pressure is same at all sides, they cancel out.
The 4th step is to use equation 85 in the different directions.
Step 5 is to use conservation of mass. Here, if we use Bernoulli's equation, we will not be considering pressure (since it is atmospheric), and no gravitaty term since we are ignoring gravity. Therefore, we get

$$\frac{1}{\cancel{2}}\rho V_1^2 = \frac{1}{\cancel{2}}\rho V_2^2$$

Using this, we can write the u and w terms. Using the substitutions shown in slide 37 and 38.
The forces depend on the angle of the vane. When it is flat, it doesn't need a x nor z directional counter forces. However, when it is completely blocking the water ($\theta = 180°$), then there is no z forces, but complete x forces.

The sign convention is dependent on the surface vector and the velocity.

10/25: More momentum

Next homework is posted. Additionally, Dr. Eggleton's slides are posted. We have to consider how we take our control volumes. The midterm exam's structure and content are posted as well. There will be extra credit (2 pts) on this exam as well.

For the sign convention, inflow involves negative dot product between the velocity and the surface direction $\hat{n}$. However, for $\vec{V}\rho\vec{V} \cdot \hat{n}$, there's three terms that have direction and are multiplied. ALL of their products have to be considered to determine if it will be positive or negative. Inflows are negative. Example: If water is *falling* out of a container (i.e., velocity is down, in the negative direction), and the $\hat{n}$ is also down, then the *flow* is positve because $\vec{V} \cdot \hat{n}$ ends up becoming $- \times - = +$; when we conisder linear momentum, we have $\vec{V}\vec{V} \cdot \hat{n}$, which then becomes negative.

Pressure force only depends on the shape. We will consider only gage pressure when we do calculations for pressure forces, because there will be air pressure outside that counters inside pressure.

Linear Momentum

See slide 42/91

We have the gives: $Q = 0.6\,\mathrm{l/s}$, $d_o = 5\,\mathrm{mm}$, $d_i = 16\,\mathrm{mm}$, $m_n = 0.1\,\mathrm{kg}$, and $P_1 = 464\,\mathrm{kPa}$. We will mamke the control volume simpler by making the CV be on the inside of the nozzle (unlike what the figure shows). The anchoring force is F_A, which is pointed up. There will be a weight force due to the nozzle, and due to the water. The pressure force at the top surface is acting down because we always consider pressure forces acting into the control volume. The downward velocity is accelerating due to gravity.
Our time derivative is

$$\cancel{\frac{\partial}{\partial t}\int_{CV} V\rho\,\mathrm{d}\forall}^{\,steady} + \int_{cs} \vec{V}\rho\vec{V} \cdot \hat{n}\,\mathrm{d}A = \Sigma F$$

We don't need to worry about integrating on the left and right side. We only consider the top and bottom surface, 1 and 2 respectively. When doing momentum, another equation we consider is the conservation of linear momentum. Using conservation of mass, we can compute the velocities. We have to use the formula for volume of a cone. If we have any questions like these, we will be given the formula. There are many ways to draw the control volume. Select the shape that produces the least unknowns/complexity.

We treat the control volume as a black box. We don't consider internal effects, unless it effects the entire control volume. In the bent pipe example, we don't cosider what happens on the inside; just the outside, which is why there is an anchoring force.

10/27: More momentum

The professor will send a notification to students who are at risk of failing the course. For exam 1, the professor may add the first quiz's points to this. The exam will on the Monday after.

In previous class, we went over example of the nozzle, and the U-bend pipe. We were also given slides to read, numbers 1 to 9. The pressure forces only depend ond the geometry of the control volume. The summation of the forces in the pipe problem requires proper sign convention. V_1 is positive, but it is inflow, hence it is negative. V_2 is outflow. We also have to satisfy mass conservation. The mass flow rate is obtained using the water density, the known velocity, and known area. The $\dot{m}(v_1+v_2)$ term is due to change in linear momentum.

Linear Momentum in a Pipe

Slide 55 of 91 in chapter 5 slides

Friction and drag forces always work in the opposite direction to flow. The pressure forces will be in opposite directions, i.e., into the control surface, at both inlet and outlet. In both inlet and outlet, the flow does not change with time. The mass flow rate is negative if it is inflow. To find the density at both location, we will use the $P = \rho RT$ equation. We will not be actively using English units.

Thrust Example

Slide 61 of 91

The jet engine is just fixed to the ground. Nevermind, it's a quiz.

10/30: Finishing linear momentum

Grade projection is mostly B. The HW problems are worth the same regardless of the number of questions in those. The extra credit was also added. The upcoming exams can bring our grades down. The projected grades are up to date only for those who did the quiz on Friday.

The only minor problem that people did in the quiz was using gage pressure instead of absolute pressure in the equation of state. The other error was people assuming $\rho_1 = \rho_2$

Linear Momentum

Slide 58/91

In this problem, we are changing the orientation to vertical, hence now there will be additional pressure drop between the inlet (1) and outlet (2). We have weight, pressure, friction, and change in velocity. The first step is to draw the control volume, but it is already drawn for us in this case. This control volume is stationary, and the flow is steady. We only consider the z direction for flow, but we do have a radial direction r as well.
As usual, we will be writing down the conservation of linear momentum equation.

$$\int_{cs} w\rho\vec{V}\cdot\hat{n}\,\mathrm{d}A = p_1A_1 - R_z - W - p_2A_2$$

Even if we find the average velocity at the outlet, we cannot assume that it will conserve the linear momentum. Our equation becomes

$$w_1\cdot -\dot{m}_1 + \int_{A_2} w_2\rho w_2\,\mathrm{d}A_2 = p_1A_1 - R_Z - W - p_2A_2$$

We have to apply the conservation of momentum to find the equation for w_2:

$$w_2 = 2w_1(1-(r/R)^2)$$

We substitute this equation into the integral term, and get the COLM equation without integrals:

$$-w_1\underbrace{(w_1\rho\pi R^2)}_{\dot{m}} + \frac{4}{3}w_1^2\rho\pi R^2 = p_1A_1 - R_Z - W - p_2A_2$$

The derivation of the equation for the momentum at the outflow is:

$$\begin{aligned}\int_0^R \left[1-\left(\frac{r}{R}\right)^2\right]^2 r\,dr &= \int_0^R \left[1-2\left(\frac{r}{R}\right)+\left(\frac{r}{R}\right)^4\right] r\,dr \\ &= \int_0^R r\,dr - 2\int_0^R \frac{r^3}{R^2}\,dr + \int_0^R \frac{r^5}{R^4}\,dr\end{aligned}$$

The base form of the equation for the conservation of linear momentum (COLM) for a moving control volume (CV) is

$$\frac{\partial}{\partial t}\int_{cv}(W+V_{CV})\rho\,\mathrm{d}V\!\!\!/ + \int_{cs}(W+V_{CV})\rho\vec{W}\cdot\hat{n}\,\mathrm{d}A = \Sigma F \tag{87}$$

For steady flow and constant V_{CV},

$$\int_{cs}\rho\vec{W}\cdot\hat{n}\,\mathrm{d}A = 0 \tag{88}$$

and we get

$$\int_{CS} W\rho\vec{W}\cdot\hat{n}\,\mathrm{d}A = \Sigma F \tag{89}$$

In the total derivative equations:

$$\frac{DM_{sys}}{Dt} = \cancel{\frac{\partial}{\partial t}\int_{cv}\rho\,\mathrm{d}V\!\!\!/} + \int_{cs}\rho\vec{W}\cdot\hat{n}\,\mathrm{d}A = 0 \Rightarrow \int\rho\vec{W}\cdot\hat{n}\,\mathrm{d}A = 0$$

Moving CV and 2D flow

Slide 66/91

We break down our equations into the different directions x and z. Since the insides of a CV are like a black box, we do not have to consider the effects inside the CV. We apply the conservation of mass.

We'll be skipping Conservation of Angular Momentum. Our midterm comes out to here; we'll not have conservation of energy. The COE equation is

$$\dot{Q} - \dot{W} = \left.\frac{\mathrm{d}E}{\mathrm{d}t}\right|_{sys} \tag{90}$$

We merge the Reynold's Transport Theorem with the unit energy

11/01: Finishing Chapter 5

The conservation of energy, covered in last class, will not be in our midterm. Heat transfer going into the system is positive. Work done by the surroundings to the system is positive. Therefore,

$$\frac{DE}{Dt} = \int_{cv} e\rho\,\mathrm{d}V\!\!\!/ = \Sigma\dot{Q}_{in} + \Sigma\dot{W}_{in} \tag{91}$$

The equation for energy is the sum of internal energy, kinetic energy, and potential energy:

$$e = u + \frac{V^2}{2} + gz \tag{92}$$

The rate of work is also known as power. For shafts, which we will be looking at, the work is

$$\dot{W}_{shaft} = T_{shaft}\omega \tag{93}$$

It can also be due to compression and shearing:

$$\Sigma\dot{W}_{in} = \dot{W}_{pressure} + \dot{W}_{shear} + \dot{W}_{shaft} \tag{94}$$

Like before, it is important to have the control volume perpendicular to the flow. The pressure work is given as:

$$\dot{W}_{pressure} = \int_{cs} -p\vec{V}\cdot\hat{n}\,\mathrm{d}A \tag{95}$$

Note that

$$h = u + \frac{p}{\rho} \tag{96}$$

The final equation becomes

$$\frac{\partial}{\partial t}\int_{cv} e\rho\,\mathrm{d}V + \int_{cs}\left(h + \frac{V^2}{2} + gz\right)\rho\vec{V}\cdot\hat{n}\,\mathrm{d}A = \Sigma\dot{Q}_{in} + \Sigma\dot{W}_{shaft} \tag{97}$$

Most of our examples are cyclical, so it will be steady (hence the $\frac{\partial}{\partial t}$ term is zero). In nearly all of our problems, we will have one inlet and outlet. There will not be a lot going in the geometry. The continuity equation can be used. The equation we get is

$$\dot{m}\left[h_{out} - h_{in} + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \Sigma\dot{Q}_{in} + \Sigma\dot{W}_{shaft,in} \tag{98}$$

Energy — Power Pump

See slide 82/91

The steps needed to do are:

1. Define the control volume
2. Write the equation for energy and Reynold Theorem, as shown in equation 97.
3. Determining the volumes and mass flow rates.
4. Substitute the equations together

The equation for flow is

$$Q = A_1V_1 = A_2V_2$$

This example uses pounds for mass, but we won't be using that much in the homework problems. For any conversion if it does arrive, we'll be provided that.

Energy - Turbine Power

See slide 87/91

The flow is adiabatic, meaning no heat transfer. There's no change in elevation.

$$\frac{\dot{W}_{in}}{\dot{m}} = \frac{\frac{\mathrm{d}W}{\mathrm{d}t}}{\frac{\mathrm{d}m}{\mathrm{d}t}} = \frac{W}{m} = w$$

In this case, the work is going out.

11/03: Midterm Prep

Today there will be nothing new. We will only be going over the exam review. The extra credit question is due Monday; we have to come up with the Reynold's Transport Theorem. The professor is looking for something like the diagrams shown in slide 54 of *Chapter 4 slides*. We have to show the derivation just like in slide 55. This is a lot of work, but it is worth 5% of the final grade.

For conceptual questions, we have to have an understanding of the material. We have to know about streamlines, unsteady and collective effects, material derivative, the acceleration field, etc.

If we get gasses, we will use $\rho = \frac{p}{RT}$ at most. The equation for flow will not be as complicated as the one in question 6 from HW6.

Mass flow rate generally is consistent: $\dot{m}_1 = \dot{m}_2$. The average mass flow rate is $\dot{m} = \rho A\bar{V}$. In the review problem 2, the average velocity is

$$\bar{V}_2 = \frac{V_{min} + V_{max}}{2} = 43.5\,\frac{\mathrm{m}}{\mathrm{s}}.$$

For question 6 from the homework, we have the option to do streamlines control volume or a boxed control volume. In the case where we use a streamline control volume (i.e., the inflow surface is small, and outflow surface is big, since there is less velocity in the outflow).

We first use conservation of mass to determine h, the height of the control surface for the inlet side.

$$\begin{aligned}
\int_{cs} \rho\vec{V}\cdot\hat{n}\,\mathrm{d}A &= 0\\
-\rho hwU + \rho\int_{(2)}\left[-2U\left(\frac{y}{a}\right)^3 + 3U\left(\frac{y}{a}\right)^2\right]\mathrm{d}yw &= 0\\
-\rho hU - 2U\rho\left[2\int_0^a \frac{y^3}{a^3}\,dy\right] + 3U\rho\left[2\int_0^a \frac{y^2}{a^2}\,dy\right] &= 0\\
-\rho hU - 4U\rho\left[\frac{y^4}{4a^3}\right]_0^a + 6U\rho\left[\frac{y^3}{3a^2}\right]_0^a &= 0\\
-\rho hU - 4\rho U\cdot\frac{a}{4} + 6\rho U\cdot\frac{a}{3} &= 0\\
-h - a + 2a &= 0\\
\therefore h &= a
\end{aligned}$$

Now that we know the h, we can use COLM by breaking it ito two sections.

$$\begin{aligned}
\int_{cs} u\rho\vec{V}\cdot\hat{n}\,\mathrm{d}A &= \Sigma F_x\\
(-\dot{m}_1)U + \dot{m}_2 u &= -F_D\\
\rho U^2 hw + \rho 2w\int_0^a u^2\,dy &= -F_D\\
\rho U^2 h + 2\rho\int_0^a\left[-2U\left(\frac{y}{a}\right)^3 + 3U\left(\frac{y}{a}\right)^2\right]^2 dy &= -F_D
\end{aligned}$$

We can use our calculators to do the final calculation for the integral.

11/08: Differential Analysis

In chapter 5, we used control volume analysis, which is a black box. Differential analysis is the opposite, where we look at the small components. Control volume analysis is an integral analysis, whereas differential analysis involves differentials (infinitesimal systems).

We have to consider how a differential element changes. Its general motion can be broken down to

- translation (regular motion)
- linear deformation (scaling)
- rotation
- angular deformation

We will continue to use the acceleration field equations. In translation, the u and v components of velocity is the same throughout the element. Since this is uniform, it does not create any deformation.

When the velocity is different at different points of the differential, we have linear deformation. Suppose one end of the differential has a velocity of u, and the velocity gradient is $\frac{\partial u}{\partial x}$, then the point on the other side will have a velocity of $u + \frac{\partial u}{\partial x}\delta x$. The amount of linear stretching is $\frac{\partial u}{\partial x}\delta x \delta t$. We are assuming that the linear deformation is in one direction. Since the volume of an element is $\delta V\!\!\!/ = \partial x \partial y \partial z$, then the change in volume in

$$\text{change in } \delta V\!\!\!/ = \left(\frac{\partial u}{\partial x}\delta x\right)(\delta y \delta z)(\delta t) \tag{99}$$

Note that δ symbolizes a small element, infinitesimally small. Δ, in contrast, indicates the change. The rate of change of the volume in linear deformation is

$$\frac{1}{\delta V\!\!\!/}\frac{\mathrm{d}\delta V\!\!\!/}{\mathrm{d}t} = \frac{\partial u}{\partial x} \tag{100}$$

For a general 3D case wehere there may be differences in velocities in any direction, the rate of change of the volume element is

$$\frac{1}{\delta V\!\!\!/}\frac{\mathrm{d}\delta V\!\!\!/}{\mathrm{d}t} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla \cdot \vec{V} \tag{101}$$

This causes a change in density. If we consider our fluid to be incompressible, then

$$\nabla \cdot \vec{V} = 0. \tag{102}$$

Rotation occurs when there is a non-uniform velocity, with respect to a *different* axis. In linear deformation, the x component of velocity changed with respect to x: $\frac{\partial u}{\partial x}$. In contrast, rotation has a change in velocity with respect to a different axis; for example: $\frac{\partial u}{\partial y}$. The angle that the surfaces rotate are $\delta\alpha$ and $\delta\beta$. The angles can be computed by trigonometry. We get

$$\delta\alpha = \frac{\partial v}{\partial x}\delta t \tag{103}$$

and the angular velocity for small angles is

$$\omega_{OA} = \frac{\partial v}{\partial x}. \tag{104}$$

A small angle is going to be our case most of the time, and this would be less than 10°. We use the right hand rule to determine the sign. If we have an angular velocity being caused by two different velocities (e.g., in the x and y directions), then the angular velocity is their average. The angular velocity vector is

$$\vec{\omega} = \frac{1}{2}\left(\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{\imath} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}\right) \tag{105}$$

which can also be written as

$$\vec{\omega} = \frac{1}{2}\mathrm{curl}\vec{V} \tag{106}$$

From this, the vorticity is defined as

$$\zeta = 2\omega = \nabla \times \vec{V} \tag{107}$$

During rotation, elements can have angular deformatio. The rate of shearing strain is

$$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}. \tag{108}$$

The derivation for the continuity equation is shown in slides 15 to 17. In general, the COM equation in differential form is

$$\frac{\partial \rho}{\partial t}\delta V\!\!\!/ + \frac{\partial \rho u}{\partial x}\delta V\!\!\!/ + \frac{\partial \rho v}{\partial y}\delta V\!\!\!/ + \frac{\partial \rho w}{\partial z}\delta V\!\!\!/ = 0 \tag{109}$$

This is similar in polar coordinates, however there will be r terms appearing.

We will talk about the stream functions in next class.

The project due date is now December 3rd.

11/10: Continuing Differential Analysis

The exam and previous homework was handed back. The average of the exam was 30% higher than previous.

For problem 2, it is conservation of mass. Section 1 was the nonuniform distribution in flow. $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$. Many people made the mistake of having the inlet/outlet size wrong. If we solved this problem, we'd get velocity at section 3 to be $-3.\bar{3}\,\mathrm{ft/s}$.

For problem 3 of the exam, we were supposed to rotate our coordinate system. We would get 3 points if we didn't flip the system and just had the equations.

$$\int_{cs} \vec{W}\rho\vec{W}\cdot\hat{n}\,\mathrm{d}A = \Sigma F$$

$$\begin{aligned} W_i &= V_j - V_{cv} \\ &= 40\,\mathrm{m/s} - (-5\,\mathrm{m/s}) \\ &= 45\,\mathrm{m/s} \\ F_A &= \vec{W}\rho\vec{W}\sin 30^\circ \frac{\pi D_j^2}{4} \\ &= 0.8769\,\mathrm{N} \end{aligned}$$

Most people got problem 4. The nozzle-elbow combination was horizontal, hence gravitational forces could be ignored.

$$\begin{aligned}
\int_{cs} \vec{V}\rho\vec{V}\cdot\hat{n}\,\mathrm{d}A &= \Sigma F \\
u_1(-\dot{m}_1) - u_2(\dot{m}_2) &= -F_{Ax} + p_1 A_1 \\
\dot{m}_1 &= \dot{m}_2 \\
&= 1000\,\mathrm{kg/m^3} \times 2\,\mathrm{m/s} \times \frac{\pi(0.3\,\mathrm{m})^2}{4} \\
&= 141.32\,\mathrm{kg/s} \\
A_1V_1 &= A_2V_2 \\
\Rightarrow V_2 &= \frac{A_1}{A_2}V_1 \\
&= \frac{300^2}{160^2}2 \\
&= 7.03\,\mathrm{m/s} \\
&- 2\,\mathrm{m/s}\times 141.37\,\mathrm{kg/s} - 7.03\,\mathrm{m/s}\times 141.37\,\mathrm{kg/s}\ldots \\
&= -F_{Ax} + 100\,\mathrm{kPa}\times\frac{\pi(0.3\,\mathrm{m})^2}{4} \\
F_{Ax} &= 8345.35\,\mathrm{N}
\end{aligned}$$

We will have extra credit quiz on Monday November 20th. We will likely not have class on the Friday after since the professor will be out for a conference. (These are likely, but not set yet).

In last class, we started going through the differential analysis. We compared it with integral method. We have to use Taylor series expansion. The net mass flow rate in the x direction, by subtracting the flow in either sides of an element, is

$$\rho u\Big|_{x+\frac{\delta x}{2}} - \rho u\Big|_{x-\frac{\delta x}{2}} \tag{110}$$

If the flow is steady, but compressible (i.e., does not change with time, but does with velocity), then our equation is

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0 \tag{111}$$

while if it is incompressible, it is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla\cdot\vec{V} = 0 \tag{112}$$

The stream function ψ is a means to reduce the continuity equation. For a 2D flow, there is no w (z) component. Therefore, u and v can be related by the stream function $\psi(x, y)$:

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \tag{113}$$

The differential force $\delta F = \delta ma$. There are many forces; we will consider weight as the body force, while pressure and friction is the surface force. We split the bod force in x, y, and z directions.

$$\delta F_{bx} = \delta m g_x = \rho g_x \delta x \delta y \delta z \tag{114}$$

is the body force in a given direction (x in this case).

When denoting stress, we use the first subscript to denote the direction of normal plane, and the second for the direction of stress. Hence, τ_{xy} is shear stress that lies on the plane whose normal is in the x direction, and y is the direction that this is going towards.

The net force on an element is simply the differential some of the forces in all the directions, shown in slide 27 of 56.

11/13: Stream Functions

In last class, we went over the forces on an element. After obtaining the forces, we obtain the equation of motion. The sum of forces of the body and surface force are equal to

$$\delta F_x = \delta F_{bx} + \delta F_{sx} = \delta m a_x \tag{115}$$

The body force is given by $\rho g_x \delta V$, while the surface force is given by

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \tag{116}$$

(these are for the x direction, but would be similar in other directions as well.

In an inviscid flow, there is no shearing stress. In that case, we will remove all the τ terms. Since pressure is applied equally in all directions, the stress is equal to the pressure: $-p = \sigma_{xx} = \sigma_{yy} = \sigma_{zz}$. This yields the equation of a form

$$\rho g_x - \frac{\partial p}{\partial x} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) \tag{117}$$

In shorthand notation, it is

$$\rho g - \nabla p = \rho\left[\frac{\partial V}{\partial t} + (V\cdot\nabla)V\right] \tag{118}$$

If a flow is irrotational, $\omega = 0$. A simple example of this is uniform flow.

Vorticity

Slide 38 of 56 in chapter 6 slides

$$V = (x^2 - y^2)\hat{\imath} - 2xy\hat{\jmath}$$

Here, the $\hat{\imath}$ term in the u component, $\hat{\jmath}$ term is the v component, and the $w = 0$. The ω_x and ω_y derivatives are zero since they involve the z direction (which is absent here). The only one we have left is

$$\begin{aligned}\omega_z &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ &= \frac{1}{2}\left(-2y - (-2y)\right) \\ &= 0\end{aligned}$$

Therefore it is irrotational.

Continuity Equation

Slide 39 / 56

Our density does not change with time. Since in the equation

$$\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

the ρ con never be zero, the inner term has to be zero. Lookig at the equations (see slides), the $\frac{\partial u}{\partial x} = 2x$, while $\frac{\partial v}{\partial y} = x + z$. Substituting these back into the previous equation, we have

$$\begin{aligned}2x + x + z + \frac{\partial w}{\partial z} &= 0 \\ \frac{\partial w}{\partial z} &= -(3x + z) \\ \Rightarrow \int \mathrm{d}w &= \int (3x + z)\,\mathrm{d}z \\ \Rightarrow w &= -3xz - \frac{z^2}{2} + f(x, y)\end{aligned}$$

Note that we are using $f(x, y)$ instead of C because we are taking the integral of $\mathrm{d}z$, which considers x and y also as constants.

Stream function

See slide 41 / 56

The stream function is a means of reducing the number of variables. We will get $u = \frac{\partial \phi}{\partial y}$and $v = -\frac{\partial \phi}{\partial x}$. We will have to integrate the two in terms of each other:

$$\begin{aligned}\int \mathrm{d}\psi &= \int u\,\mathrm{d}y \\ \Rightarrow \psi &= \int 2y\,\mathrm{d}y \\ &= y^2 + f(x)\end{aligned}$$

and we will do the same for the v term:

$$\begin{aligned}\int \mathrm{d}\psi &= -\int v\,\mathrm{d}x \\ \Rightarrow \psi &= -\int 4x\,\mathrm{d}x \\ &= -2x^2 + f(y)\end{aligned}$$

If we combine the two equations, we can get the $f(x)$ and $f(y)$ term (they are literally just each other):

$$\psi = -2x^2 + y^2 + C$$

We still have the constant C because we don't know if the $f(x)$ or the $f(y)$ term had a regular constant or not. However, we can still draw the streamline by assigning different values for C.

The velocity potential is $\phi(x, y, z, t)$. We substitute the $\vec{V} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$ definition with

$$\vec{V} = \frac{\partial \phi}{\partial x}\hat{\imath} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial t}\hat{k} \quad (119)$$

$$= \nabla\phi \quad (120)$$

$$\nabla \cdot \vec{V} = 0 \quad (121)$$

$$= \nabla(\nabla\phi) \quad (122)$$

$$\nabla^2\phi = 0 \quad (123)$$

Velocity potential and Inviscid Flow Pressure

Slide 42 / 56

We are given that this is a horizontal plane (no z term). The components in the stream function are

$$
\begin{aligned}
\psi &= 2r^2 \sin 2\theta \\
v_r &= \frac{1}{r}\frac{\partial \psi}{\partial \theta} \\
&= \frac{\partial \phi}{\partial r} \\
v_\theta &= -\frac{\partial \psi}{\partial r} \\
&= \frac{1}{r}\frac{\partial \phi}{\partial \theta} \\
v_r &= \frac{1}{r}\left(2r^2 \cos(2\theta) \cdot 2\right) \\
&= 4r\cos(2\theta) \\
v_\theta &= -\frac{\partial \psi}{\partial r} \\
&= -4r\sin(2\theta) \\
v_r &= \frac{\partial \phi}{\partial r} \\
\Rightarrow \int \mathrm{d}\phi &= \int 4r\cos(2\theta)\,\mathrm{d}r \\
\Rightarrow \phi &= 2r^2\cos(2\theta) + f(\theta) \\
v_\theta &= \frac{1}{r}\frac{\partial \phi}{\partial \theta} \\
\Rightarrow \phi &= -\int 4r^2 \sin(2\theta)\,\mathrm{d}\theta \\
&= 2r^2\cos(2\theta) + f(r) \\
\therefore \phi &= 2r^2\cos(2\theta) + C
\end{aligned}
$$

11/15: Potential Flow and Navier-Stokes

Don't bother showing up on Friday. Third extra credit quiz will be from chapter 5 on Monday. He will prepare some homework. The next HW will be due after Thanksgiving. His office hour today will be from 1 to 2.

In last class, we went over example 4. Since we know initials, we can do integral when trying to compute ϕ. If we plotted a sin and a cos curve, they would be normal to each other at every point of intersection.

Four examples of potential flow that satisfy incompressibility and irrotationality are: uniform flow (flow goes in one direction only), source or sink (flow radially going outward or inward from a single point), free vortex (flow moving around a single point), a doublet (one source and sink; the space in between is a doublet; we will not cover this but is available in the book).

In uniform flow, velocity is in only one direction. If it flows in the x direction only, then $U = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$.

$$
\frac{\partial \phi}{\partial x} = U \to \int \mathrm{d}\phi = \int U\,\mathrm{d}x \to \phi = Ux + C
$$

In a source or sink, the v_r cannot be constant because it would not satisfy the conservation of mass. $A_1V_1 = A_2V_2$; if 2 was the outer radius and 1 was inner, $A_2 > A_1$, hence $V_2 < V_1$. The velocity is therefore given as

$$
v_r = \frac{m}{2\pi r} : v_\theta = 0 \tag{124}
$$

If $m > 0$, it is a source, otherwise a sink.

$$
\begin{aligned}
\frac{m}{2\pi r} &= \frac{\partial \phi}{\partial r} \\
\int \mathrm{d}\phi &= \int \frac{m}{2\pi r}\,\mathrm{d}r \\
\phi &= \frac{m}{2 = pi}\ln r
\end{aligned} \tag{125}
$$

And the stream function is

$$
\begin{aligned}
v_r &= \frac{1}{r}\frac{\partial \psi}{\partial \theta} \\
\Rightarrow \mathrm{d}\psi &= \frac{m}{2\pi r} r\,\mathrm{d}\theta \\
\psi &= \frac{m}{2\pi}\theta
\end{aligned} \tag{126}
$$

A free vortex is similar, except that it goes round a singularity rather than radial to the singularity. The streamlines will be concentric circles. At the center, the velocity is also near infinity.

$$
\begin{aligned}
\phi &= K\theta \\
v_\theta &= \frac{1}{r}\frac{\partial \phi}{\partial \theta} \\
&= \frac{1}{r}K \\
\Rightarrow \psi &= -K\ln r \\
v_\theta &= -\frac{\partial \phi}{\partial r} \\
&= \frac{k}{r}
\end{aligned} \tag{127}
$$

Note that this is not the same as rigid body rotation. In rigid body rotation, the whole thing rotates together with the same angular velocity; this would not be a irrotational flow. In contrast, if the velocity is highest at the center and lowest near the edges, that would be a potential flow. In rigid body rotation:

$$
\omega_z = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \neq 0 \tag{128}
$$

while for free vortex, it is

$$\omega_z = \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = 0 \tag{129}$$

Another way to look at free vortex flow is to look at contours Γ or circulation. This is useful because we can compare the different potential flows. See slide 50 of 56.

In viscuous flow, there is shear stresses. After a lot of substitutions and cancellations, the Navier-Stokes equation is obtained:

$$\rho\left(\frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}\right) = -\nabla p + \rho\vec{g} + \mu\nabla^2\vec{V} \tag{130}$$

This is extremely difficult to get a solution of.

In steady laminar flow between plates, we can cancel a lot of terms as shown in slide 52 of 56. These cancellations can be done because of single direction of gravity, and flow in a single direction. The new equations obtained are:

$$0 = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{131}$$

$$0 = -\frac{\partial p}{\partial y} - \rho g \tag{132}$$

$$0 = -\frac{\partial p}{\partial z} \tag{133}$$

Notice that the second equation is just the hydrostatic equation, and the third one is obvious. The first equation is the interesting one:

$$\begin{aligned} 0 &= -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2}\right) \\ \Rightarrow \frac{\partial^2 u}{\partial y^2} &= \frac{1}{\mu}\frac{\partial p}{\partial x} \\ \Rightarrow \frac{\partial u}{\partial y} &= \frac{1}{\mu}\frac{\partial p}{\partial x}y + c_1 \\ \Rightarrow u\frac{1}{2\mu}\frac{\partial p}{\partial x}y^2 &+ c_1 y + c_2 \end{aligned}$$

The $\frac{\partial u}{\partial y}$ is the slope of the parabolic curve of the flow front of the flow passing between the parallel plate.

We will continue with examples of these next time.

11/22: Cylindrical Flow

There was a quiz about our favorite equation.

The most important part of potential flow is that the governing equations $\nabla^2\psi = 0$ and $\nabla^2\phi = 0$ are linear. Therefore, they can be superimposed. For example, if we have both a source and a sink to get a doublet, we simply add the two stream functions

$$\phi_1 + \phi_2 = \phi_{doublet}$$

The equation for a doublet is

$$\psi_{doublet} = -\frac{K\sin\theta}{r} \tag{134}$$

with the full derivation shown in the book.

When there is an uniform flow in addition to a source, there will be a stagnation point between the uniform flow and source. The program MFM2 can be used to visualize some of these stuff. Rankine half bodies are with uniform flow and source; Rankine full bodies additionally include a sink after the source.

When the sink and source come together to form a doublet while beinng in uniform flow, we'll get cylinder flow. On the surface of this cylinder, there will only be angular velocity of fluid, no radial. The strength of the doublet will be $K = Ua^2$. Since we are assuming inviscid flow, at surface of cylinder, we'll get tangential flow. Where we have maximum velocity, we have minimum pressure.

Theoretically, a rankine full body should not have any drag. However, experimental results show that there will be drag. This is called the d'Alembert's paradox. We assumed inviscid in theory, but there will be viscosity. The friction is responsible for the drag.

If there is uniform flow, with doublet, and a free vortex, it will be cylinder flow with lift. $F_y = -\rho U\Gamma$. We can get drag if we consider viscid flow. This is called the Kutta-Jukowski theorem. The pressure difference will be non-symmetrical.

In homework question 7, we'll need 4 boundary equations. The two fluids' velocities are identical at the boundary layer between them. The fourth equation will be the shear equation.

For homework question 8, the free surface that is in contact with air has no shear stress; it will flow freely.

The slides on BlackBoard will be updated to include the cylindrical stuff.

11/27: Buckingham Pi

Today we start with chapter 7. Extra credit quiz results were returned.

Flow similarity is when we compare two different flows if they have similar characteristics. We can get this by non-dimensionallizing certain parameters. These are important because it allows us to make, for example, a small aircraft to test, and then use flow similarity to make a bigger airplane.

Dimensional analysis is going from dimensions and determining the non-dimensional numbers. Some non-dimensional numbers are Reynold's number.

Often, it is used to make small scale models represent larger models. When modelling an airplane, for example, we can scale the model by $\frac{1}{3}$ and increase the density by 3, so that the overall Reynold's number remains unchanged:

$$\text{Re} = \frac{\rho v L}{\mu}$$

The pressure drop is related with the diameter of the pipe, the density, the viscosity, and velocity. We can do different experiments such as finding pressure drop as we change the velocity, diameter, density, or viscosity. However, these experiments are unrealistic, complex, and expensive. As an example, changing the viscosity likely will change density. Hence, we have to convert the equation.

The Buckingham Pi Theorem is used for this.

1. List all variables in the problem.
2. Express the variables in terms of the fundamental dimensions.
3. The number of pi numbers $N = k - r$, where k is the number of variables, and r is the number of most basic dimensions used. In the cause of the pressure drop equation, $k = 5, r = 3 \implies N = 2$.
4. Select a number of repeating variables and non-repeating variabales that we want to represent in
5. Form a Π term by multiplying the non-repeating variable with repeating variables until everything becomes zero.

$$\begin{aligned}
\Pi_1 &= \Delta p_{\updownarrow} D^a V^b \rho^c \\
(FL^{-3})^1 (L)^a a (LT^{-1})^b (FL^{-4}T^2)^c &= F^0 L^0 T^0 \\
F: 1 + c &= 0 \\
L: -3 + a + b - 4c &= 0 \\
T: -b + 2c &= 0 \\
a &= 1 \\
b &= -2 \\
c &= -1 \\
\Pi_1 &= \frac{\Delta p_{\updownarrow} D}{\rho V^2}
\end{aligned}$$

6. Repeat the previous step for the other nonrepeating variables. We get Π_2 for μ (see chapter 7 slides).
7. Check all resulting terms are dimensionless and independent. Being independent means that they cannot be re-written one in term of each other. For example, if there was a problem that involved d diameter and h height, both of these are length units, hence cannot be used as the non-repeating variable.

Example 1

See example 1 of chapter 7 slides

Π_1 will be for $D = f(w, v, \rho)$:

$$\begin{aligned}
\Pi_1 &= (MLT^{-2})^1 (L)^a (LT^{-1})^b (ML^{-3})^c \\
&= M^0 L^0 T^0 \\
M: 1 + c &= 0 \\
\Rightarrow c &= -1 \\
L: 1 + a + b - 3c &= 0 \\
\Rightarrow b &= -2 \\
T: -2 - b &= 0 \\
\Rightarrow b &= -2 \\
\therefore \Pi_1 &= \frac{D}{w^2 v^2 \rho}
\end{aligned}$$

For Π_2 for $h = f(w, v, \rho)$, we can do this by inspection:

$$\begin{aligned}
\Pi_2 &= h w^a v^b \rho^c \\
\Pi_2 &= \frac{h}{w}
\end{aligned}$$

Example 2

See example 2 of chapter 7 slides

Since four variables are dependent on L, we can only use 1 of them (let's select D). Then we select another variable that has different units, for example, E. Notice that we can get E by $E = \gamma D$. For the first Π term, we get

$$\begin{aligned}
\Pi_1 &= \delta D^a \gamma^b \\
&= \frac{\delta}{D}
\end{aligned}$$

And for E, we have

$$\begin{aligned}
\Pi_2 &= E D^a \gamma^b \\
&= \frac{E}{D\gamma}
\end{aligned}$$

Geometric similarity is where the ratios of the size of the actual model and testing model are the same. Dynamic similarity is where the real and prototype model have identical flow conditions. Kinematic similarity is when the streamlines look similar.

Space shuttle example was given.

This concludes chapter 7.

11/29: Boundary layers and airfoils

We will be skipping most of chapter 8. Chapter 9 may be our last chapter.

If we had a very thin flat plate with viscous flow, and our Reynold number is less than 1, then viscous forces dominate. The flow ahead, above, below, and behind the plate will be affected.

As the Reynold's number is increased, the viscuous effects are reduced. At high Reynold's number, such as 10000000, the boundary layers form at the leading edge. Outside the boundary layer, the velocity is U, the upstream velocity. The thickness of the boundary layer (distance between plate surface and the boundary layer) increases as we move in the direction of the flow.

The boundary layers can be laminar or turbulent. In fact,

$$\text{Re} = \frac{\rho u x}{\mu} \tag{135}$$

where x is our location from the leading edge of the plate. Hence, the Reynold's number increases as we move to the trailing edge, and hence dictates whether the flow is laminar or turbulent.

In the inviscid region, we can assume that the flow is straight. When flow gets into the viscid region, it will start shearing (if in laminar region). When there's full turbulence, all sorts of things can happen.

We can describe the boundary layer with its thickness δ^*. The δ region is from where $u = 0$ all the way to the boundary where $u = 0.99U$. There will be a deficit in mass flow rate; we can look it instead as having a constant velocity $u = U$ everywhere from 0 to δ^*, where $\delta^* < \delta$.

$$\delta^* = \int_0^\infty 1 - \frac{u}{U}\, dy \tag{136}$$

A similar idea is momentum thickness, where we consider the momentum instead. Use use the equation $u\dot{m} = u^2 \rho A$.

In the turbulent section, the flow is dictated by many eddies. Here, there is an increase in shear stress.

Prandtl was the guy who solved a lot of paradox, and came up with the boundary layer problem. Blasius was his student, and he came up with a solution for the boundary layers. Blasius introduced some similarity variables and came up with the solution

$$\delta = 5\sqrt{\frac{vx}{U}} \tag{137}$$

which tells us how much the boundary layer thickness is based on our position x, velocity (upstream) U, and the kinematic viscosity of the flow v.

There are other estimates, and all are pretty similar.

In pipes, laminar is if Reynolds number is less than 2100, and turbulent if more than 4000. The flow becomes fully developed after some length $\updownarrow_e$.

The pressure gradient in the x region is constant once flow is developed. This is true for parabolic pressure gradients. The shear stress is highest at the walls of the pipe, and 0 at the centerline.

There is high pressure in the front and bottom of an airfoil; low pressure at the top. Shear stress somehow always acts on the wall of the airfoil. These results in aerodynamic forces lift and drag.

One way to calculate lift and drag is to take the integral of the forces in the x and y. We don't know the angles everywhere. Also, even if compute the pressure, computing the shear stress is much much harder.

In example 1 of chapter 9 slides, the plate generates no lift because the top and bottom have the same pressure. There is, however drag due to shear stress. Now, if the flat plat was vertical (hence perpedicular to the flow), then shear stresses cancel out (remember that this is a thin plate). There will be higher pressure on the center of the plate and less pressure on the right side of the plate, hence drag is created. However, if we used a different angle between 0 and 90 (non inclusive), then it will have inequal shear and forces, hence generate lift.

For simple geometries, we can make lift and drag equations for simple geometries. This is not possible for most airfoils; for most real airfoils, the lift and drag coefficients are empirically measured. For the lift and drag equations, we use the *planform* area A.

The frictional drag depends mostly on the roughness of the surface, the orientation, and Reynolds number. The pressure drag is due to pressure distribution, and is called the form drag. We need to add up the frictional and pressure drag to obtain the total drag.

For a sphere or cylinder for example, as Reynolds number increases, the coefficient of drag decreases; thtere is a dip in the coefficient when flow turns turbulent. Different geometries have different behavior.

The homework is pushed to next Monday.

12/01: Airfoils, Lift, Drag, Stall

Our exam will be on Friday the 15th. We will likely have a quiz next week. We will have an exam review on Monday the 11th. The homework will be due on Monday the 4th. Another homework will be due on 11th. The professor will count 7 homework. If we do the extra homework (homework 8), that will count as extra credit. We should treat these homework as a review for the final. The final exam structure is not known yet. There is no minimum or maximum length of the project. We can have the project be a word document or a poster or a video or a presentation. The format and length is open to us.

The slides for this chapter will be posted once the professor has finished completing the slides.

Lift is perpendicular to the flow, and pressure is parallel; these are caused due to shear forces and pressure. Interactions between different components increase drag. In general, adding stuff together increases drag than just the sum of the stuff as is.

At a certain Reynold's number, the drag ends up dipping down.

The thickness of the boundary layer is δ, and δ^* is the displacement. In example problem in slide pg. 24, mass is conserved.

$$\begin{aligned}
\delta^*(x) &= 0.007x^{0.5} \\
U_1 &= 10\,\text{ft/s} \\
A_1 &= 4\,\text{ft}^2 \\
\dot{m}_1 &= \dot{m}_2 \\
\Rightarrow Q_1 &= Q_2 \\
A_1 V_1 &= \int_{A_2} u(x)\,\mathrm{d}x
\end{aligned}$$

The δ^* term allows us to assume 'no flow' in one side, and complete flow in the other. The final velocity will be higher, and the area is lesser.

$$\begin{aligned}
40\,\text{ft}^3/\text{s} &= U(2\,\text{ft} - 2\delta^*)^2 \\
\Rightarrow U &= \frac{10}{(1 - 0.0070x^{0.5})^2}\,\text{ft/s}
\end{aligned}$$

In curved surfaces, velocity changes, hence pressure changes. The pressure gradient will be nonzero. If we consider flow around a cylinder, stagnation points occur at the front and back of the cylinder. Favorable pressure gradient is in the front of the cylinder since pressure decreases; in the back half, it has to gain pressure. As a result, in the back half, the flow particles may not have enough energy to regain pressure, and hence separate. In the inviscid theory, flow does not separate, but in reality it does.

If we keep increasing our Reynolds number, flow separates more, but at a point, they can start rejoining again. Compressing is easy, expanding is hard (in terms of attempting to not have flow separation); hence, when making wind turbines, the inlet section can have steep angles (compression), while the outlet section has to have very small angles (expansion).

Higher speed increases the coefficient of lift. The mean camber line is above the chord line if it generates lift. The maxim separation between mean camber and chord line is called just the camber. The NACA 4 digit nomenclature: the first digit is the max camber %, the 2nd digit is the location of the max camber %/10, and the last 2 digits is the maximum thickness in %.

A symmetrical airfoil (they start with 00xx) does not generate lift at 0 angle of attack. These are helpful for stabilizers (e.g. vertical stabilizer) that should not generate forces in perpendicular directions. Also helpful for flying upside down.

Coefficient of lift increases until a point, after which it decreases (that is when flow starts separating). That's when it stalls.

12/04: Supersonics

We have to do the course evaluation. Generally, reviews are bimodal (people either really like a product or really dislike a product). The course evaluations are anynomous, but we still could technically be found out.

A cambered airfoil has lift even at 0 degrees angle of attack. The stall is when flow separates. Aspect ratio is ratio between the chord and the wingspan. As aspect ratio increases, the coefficient of lift increases; additionally, the coefficient of drag is decreased. Ideally, we want to have a large C_L/C_D ratio; this is what sailplanes have. Normally, when counting the wingspan, we only consider the actual wing. c is the chord length, and b is the wingspan, so $AR = \frac{b}{c}$. Due to a high pressure region at the bottom and low pressure at the bottom, wing tip vortices generate: the air from the bottom circulates up to the top at the tips.

When there is angle of attack there is an induced drag coefficient. The total drag is the sum of the two:

$$C_D = c_d + C_{D,i} \text{ where } C_{D,i} = \frac{C_L^2}{\pi e AR} \tag{138}$$

One solution to prevent the vortice is to have winglets that prevent the downlash.

The pressure coefficient is

$$C_p \equiv \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}. \tag{139}$$

Technically, we could get infinite lift if we approach Mach number 1, which is not true. The above assumption only works for upto mach number of 0.7. Realistically we can go faster than mach 1.

Watched Ted Ed video on passing the supersonic booms. Supersonic booms creates a double boom (but heard as single boom): 1 for the front of the airplane, and the other at the behind. Animals can create sonic booms.

Schliren is a common flow visualization to see waves and shockwaves.

There maybe regions on the airplane that reach the shockwave first. For example, the top of the wing may have higher flow speeds. This is determined by the thickness of the airfoil. Normally, faster airplanes have thinner airfoils. Currently the SR-71 is the fastest plane (to the professor's knowledge). The shockwave creates adverse pressure gradients. After the M_{cr} point, the drag significantly increases due to flow separation up to the mach number of $M = 1$. Sometimes when we fly on a regular plane, we might actually be able to see a small flow separation on the engine. At really high altitudes, we can also see it on the airfoil.

If we want to fly faster without the flow separations, we have to find a way to push the M_{cr} point closer to 1. One method is to sweep the wing backwards. The effective mach number for a swept back is $M_\infty \cos(\theta)$. In general,

$$M_\theta = \frac{M_{cr}}{\cos(\theta)} \tag{140}$$

In a delta wing, there shape allows them to have smaller vortices.

12/06: Shockwaves

We can vote our favorite topic for the quiz here. On Friday, we will be looking at schlieren imaging, and then have the quiz. The finals would be with what we covered after the 2nd exam, but we may still have equations from the first few chapters. On Monday, we will have review. Homework 7 is optional for us to submit. Final is on Friday the 15th, 10:30 to 12:30.

We may have seen shadowgraphs in our daily lives. For example, on a hot day, we can see light deflect through the hot air. The different density gradient creates some regions of more light or less.

The Schlieren, in contrast, uses two parabolic mirrors. In German, Schlieren means 'streak'. A point light source projects on the parabolic mirror; the reflected light is parallel, and when it hits the other mirror, it bounces to the camera or the destination. A filter has to be used. When light passes through a biconvex lens, it focuses to a single point. A knife is used for the filtering.

Last class, we talked about supersonic flows. The equation of state for ideal gas is

$$p = \rho R T \tag{141}$$

where R is the ideal gas constant. The specific heat at constant pressure is

$$c_p = \left(\frac{\mathrm{d}h}{\mathrm{d}T}\right)_p \tag{142}$$

and for constant volume it is

$$c_V = \left(\frac{\mathrm{d}u}{\mathrm{d}T}\right)_v \tag{143}$$

For air, the $\gamma_{air} = 1.4$. For these kind of flows, we assume that there will be no change in entropy. Using the second law of thermodynamics, an adiabatic and frictionless flow will have constant entropy:

$$\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \frac{p_2}{p_1} \tag{144}$$

The mach number is the velocity of the object divided by the speed of sound.

$$M = \frac{V}{c} \tag{145}$$

where the speed of sound is $c = \sqrt{\gamma R T}$.

The sound waves radiate concentrically from the source. If we are moving subsonically, the source of the sound moves, but the sound waves start piling up in front of the direction of travel. At the speed of sound, all the waves are piled up in front of the direction of travel. Past mach 1, a mach cone is formed, where the sound originates behind the object. The mach angle is

$$\mu = \arcsin \frac{1}{M} \tag{146}$$

Mach number example

Aircraft cruises 1000 m high, above us. It has a mach number of 1.5. The temperature is 20°C . How many seconds after the plane passes do we hear the aircraft?

We know that the angle beween the mach cone and the ground is

$$\alpha = tan^{-1} \frac{1000\,\mathrm{m}}{Vt}$$

and the mach number is

$$M = \frac{1}{\sin\alpha}$$

and

$$V = M \cdot c$$

Using the substitutions earlier, the speed of the mach cone is 514.6 m/s, and the time is $t = 2.7\,\mathrm{s}$.

When aircrafts are moving, the lines we see are due to pressure dropping, and hence condensation forming.

If we reduce area, such as in a nozzle, the air accelerates as area decreases to obey continuity equation. Then if the nozzle expands, it decelerates. Our equation (this is basically Bernoulli's equation before applying the integral):

$$\mathrm{d}p + \frac{1}{2}\rho\,\mathrm{d}(V^2) + \gamma\,\mathrm{d}z \tag{147}$$

Using this equation (and additional derivations shown on page 65 of the slides), we find an equation for the speed of the air due to choking.

A normal shock wave is where the shockwave is perpendicular to the flow direction; this happens when we go from supersonic to subsonic. An oblique shock wave happens when one of the sides has an angle (wedge); the oblique shockwave is weaker than the normal one. Instead of a wedge, if there was an expansion angle, we'd get an expansion fan, where the shock line is angled. In general, a concave corner creates oblique shock, and a convex corner creates an expansion fan.

As a result, for a flat plate moving supersonic, expansion waves are created at the top plate, and an oblique shock wave at the bottom, in front of the plate. A supersonic aircraft has more of a diamond shape wing. For a zero degree angle of attack, the front of a diamond airfoil has oblique in the front, expansion fan at the middle, and again oblique at the end.

Ideally, we'd want a supersonic nozzle to take in air at mach 0, be mach 1 at the throat, and then exit out air that is greater than mach 1. To do this, we would need a big enough pressure ratio between the inlet and the outlet.

On a shuttle rocket, we can have shockwaves at the outlet and another later.

12/11: Review

Today's class is review. Homework 8 is optional. We can expect derivations of equations and theory on the exam. Energy equations will not be in the final, in fact, chapter 5 can be forgotten, since we had quizzes and stuff for that. We may end up using some of the equations though. Calculating drag based on wake profile will not be on the exam. We may need to use equations before chapter 5, but that will not be the *main* focus. There will be 8 questions (7 require solving).

For Buckingham Pi, for the repeating variables, we should select the ones that are not dependent on other variables. In the final, if this question comes, he will tell us which variables to use as the repeating variables. If the equation already has a dimensionless coefficient, it itself is a Π number; it should not be placed when doing the Buckingham Pi derivation.

We can expect some of our final answer to be in the form of a ratio.

There will be no syphons nor cylindrical.

Most questions will have parts. Since there is no numerical problem, mistakes shouldn't carry over.

The poll results and solutions are on BlackBoard. We should look at the slides. Chapter 8 will not be on the final.

The momentum thickness for boundary layers is

$$\Theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)\mathrm{d}y \tag{148}$$

while the displacement thickness is

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right)\mathrm{d}y \tag{149}$$

The shear stress is

$$\tau_w = \rho U^2 \frac{\mathrm{d}\Theta}{\mathrm{d}x} = \mu\frac{U}{\delta} \tag{150}$$

We are enocuraged to look at question 6 of homeworkk 5.

In the Navier Stokes equation, there is no motion in the y direction for pipe flows.